EVALUATION OF A TECHNOLOGICAL CHANGE IN PRODUCTION

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THESIS

EVALUATION OF A TECHNOLOGICAL CHANGE IN PRODUCTION

bу

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Evaluation of a Technological Change in Production

by

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ABSTRACT

The Management System Development Office (MSDO) of the Naval Air Integrated Logistic Support Center (NAILSC) is presently evaluating a prototype system at the Naval Air Rework Facility, North Island (NARFNI). The system is called "work-in-progress inventory control system", (WIPICS). This paper is concerned with that evaluation and shall utilize before WIPICS production data provided by NARFNI for analysis. Three production functions for three different outputs each with three input variables are discussed and inter-relationships are identified and explained. Three Cobb-Douglas production functions constrain the cost minimization problem. Prices of inputs are constant and a minimum budget is derived for various output levels. Finally, a discussion of the possible results and their meanings is offered to improve understanding of the before and after WIPICS analysis.



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I. INTRODUCTION

A. ORGANIZATION, MSDO

The purpose of this thesis is to provide tools and concepts for the evaluation of a new system, the Work In Process Inventory Control System (WIPICS). This evaluation is being conducted by the relatively new Management Systems Development Office (MSDO). The organizational alignment of the three year old MSDO is through the Commander, Naval Air Integrated Logistics Support Center (NAILSC), Patuxent River, Maryland to Naval Air Systems Command (NAVAIRSYSCOM), Washington, D.C., (Figure 1). Program management is provided by NAVAIRSYSCOM 105, the Management Information Systems Development Division (MISDD). MSDO is organized and staffed to provide management input as well as computer systems input in the development of an integrated Management System and Management Information System (MS/MIS).

B. ORGANIZATION, NARFNI

The Naval Air Rework Facility, North Island (NARFNI) is one of the largest repair facilities of its kind in the world. It is presently one of seven facilities servicing aircraft of the United States Navy and Marine Corps. It is directly responsible for major maintenance, incorporation of technical changes and repair of "crash damaged" aircraft for West Coast based F-4, F-8 aircraft and H-46, H-53 and H-3 helicopters, as well as engines and components contained



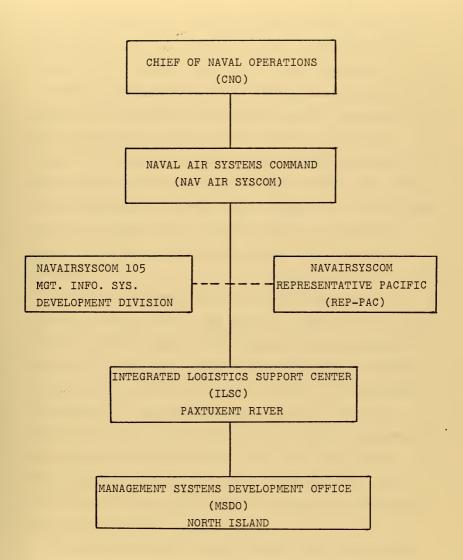
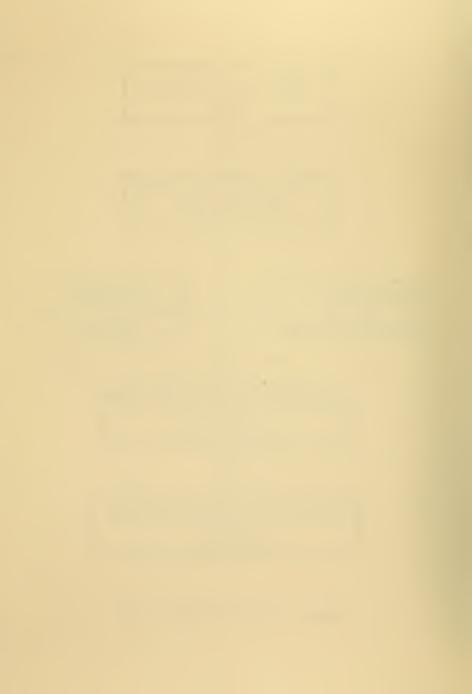


Figure 1. CHAIN OF COMMAND TO MSDO



within those vehicles. The NARF primary objective is to complete the required workload during a specific period of time at the minimum total cost to the government.

NARF is responsible to NAVAIRSYSCOM, Representative,
Pacific in carrying out its assigned tasks. Evaluation of
fleet requirements is made by the CNO and promulgated
through the Commander-in-Chief, Pacific Fleet (CINCPACFLT),
from an interpretation of the Five-Year Defense Plan (FYDP)
of the Department of Defense (DOD) [Figure 2]. The services
provided by NARFNI are an integral part of our national
defense in terms of operational aircraft and helicopter
strengths.

The large investment that the government has made in the NARFNI can be seen in the physical plant consisting of buildings, hangers test cells, laboratories, and airport facilities. The Plant, encompassing 298 acres, is valued at \$115 million, plus \$18 million budgeted for rebuilding and expansion. The yearly budget is \$150 million and approximately 300 aircraft and 100,000 related components are repaired or overhauled each year. The total labor force employed by NARFNI is nearly 6,800 persons. This force is split about half and half between supervisory and production employees.

Two major problems facing all NARF's are the variable workload and optimal utilization of manpower and material.

Each quarter of the fiscal year conferences are held with NARF representatives and CINCPACFLT representatives. NARFNI



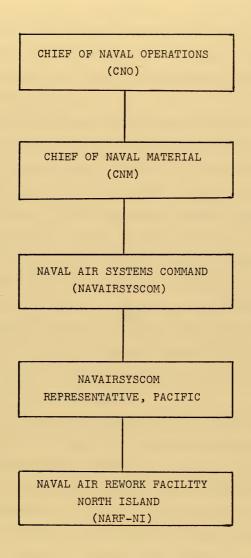


Figure 2. CHAIN OF COMMAND TO NARF-NI.



contracts, at these conferences, to rework a specific number of aircraft and components during the upcoming fiscal quarter and attempts to schedule an even flow of work. Without this scheduling the optimum utilization of resources would be more difficult.

Since all aircraft and engines utilized by the military are highly sophisticated and complex in nature, a planned program for maintenance is the key to continued performance and operational readiness. NARF provides a regular cycle of maintenance called Progressive Aircraft Rework (PAR) for each aircraft. Prior to induction in the NARF, an estimate is made, from historical data, of the man-hours and material required to rework an aircraft in order to bring it up to technical standards set by NAVAIRSYSCOM. A good example would be a four-year-old F-4J, after three cruises to the Western Pacific (WESTPAC), on the average, would require ten thousand man-hours and thirty-two days to complete the work required.

When an aircraft is scheduled for a PAR induction date, the necessary components are ordered and managers must determine the optimal allocation of existing manpower resources necessary to accomplish the workload. Simultaneously, several manpower utilization options must be feasible in case the work load is varied due to combat losses or other unforeseen changes. Aircraft are scheduled to arrive a few days prior to induction in order to be inventoried and inspected. On day zero, it enters the system and should



then take an exact number of days and consume a set number of man-hours to complete the overhaul. With this background in mind, it is easy to see that the efficient allocation of resources and manpower is no easy task. WIPICS is an attempt to make use of today's computer technology to provide NARF managers with accurate, up-to-date information as a basis for their allocation decisions.

C. WIPICS

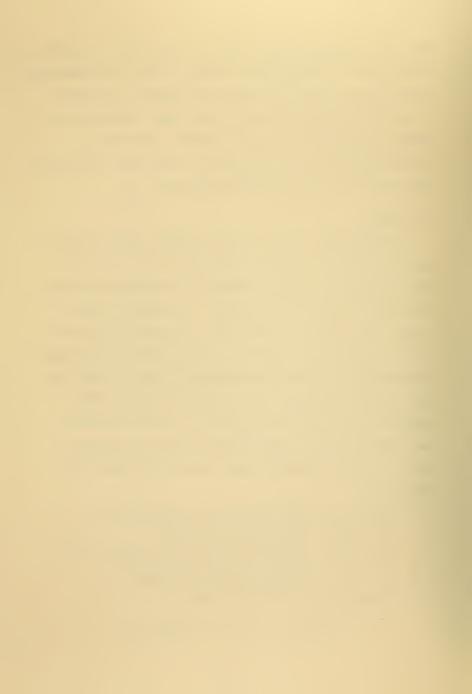
In early 1969, at the request of MSDO, ROHR Corporation studied the Naval Air Rework Facility, Naval Air Station, North Island, California (NARFNI), to investigate possible adaptation of WIPICS to the NARF. By November, 1969, a request was initiated by MSDO for a contract with ROHR to install a prototype of WIPICS. The Secretary of the Navy approved and authorized the prototype in July of 1970 and the contract negotiation phase began in October, 1970.

[Figure 3] The effective date of the ultimate contract was 4 January 1971. Prototype milestones were agreed to with a two year completion date set at 31 December 1972.

[Figure 4]

- 1. Early 1969 ROHR Corporation studied NARF-NI
- 2. Late 1969 MSDO requested contract
- 3. Early 1970 NAVAIRSYSCOM evaluation and approval
- 4. July 1970 SECNAV authority for prototype
- 5. Late 1970 Contract negotiation phase
- 6. 4 January 1971 Contract date

FIGURE 3. Historical Background



- 1. D-Day Effective date of contract 4 January 1971
- 2. D + 9.5 months WIPICS System developed
- 3. D + 11 months WIPICS Systems test
- 4. D + 11.5 months WIPICS prototype started
- 5. D + 24 months WIPICS prototype complete 31 December 1972

FIGURE 4. WIPICS Milestones

How WIPICS came into existence has been briefly explained and the external organization of the NARF and MSDO has been presented in order to better understand the part that WIPICS plays in the NARF. Now the question can be asked, Why WIPICS? WIPICS exists probably as the result of the farsightedness of one Naval Officer, Rear Admiral Smith, who clearly saw the need for more advanced management systems and then did something about it. WIPICS is an extension of NAILC/MIS Plan of 1969 and is an attempt to improve management decisions made by the officials responsible for the production of aircraft and components at NARFNI. The hoped-for production improvements will come as either a dollar savings or actual production-time savings.

WIPICS is designed to control work in process by utilizing a computer and the computer's associated memory bank. This system is designed to use any on-line computer system with minimum changes. Presently, the Rohr Corporation is using the IBM-360 as the primary unit. Future application



of the system would encompass the other \sin NARFs and each would have common programs.

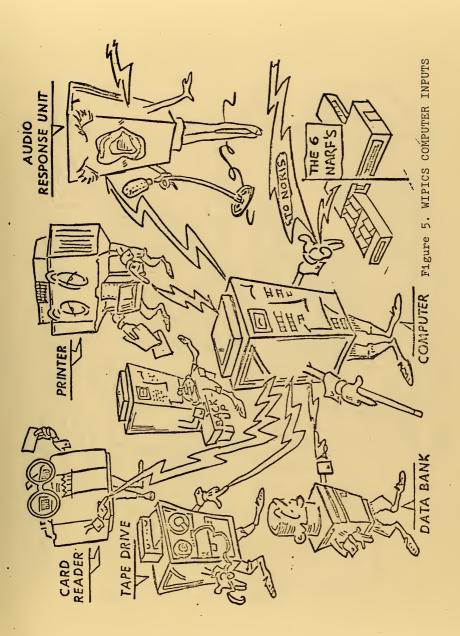
The hardware used in addition to the computer encompasses 164 touchtone and 20 alphanumeric terminals strategically placed throughout the repair facility and four teletypewriters for on-line printing of selected data as well as batch processing and printed reports of off-line information. One IBM 7770 audio response unit with a vocabulary of approximately 80 words is used for on-the-spot recall of data for information or verification. This unit is connected to all 184 phone terminals, touchtone and alphanumeric located throughout the shop. A data card reader is also available for use in batch processing of daily, weekly or quarterly information. The system in this form is capable of handling 30,000 transactions per day. Another IBM 360 is used as a back-up unit and either unit has a file dump to tape for use every 24 hours.

Inputs to this system can be made by touchtone from the plant floor, by data card through a card reader, from the data bank, by audio response unit, from the teletypewriter terminal or from a tape source. [Figure 5]

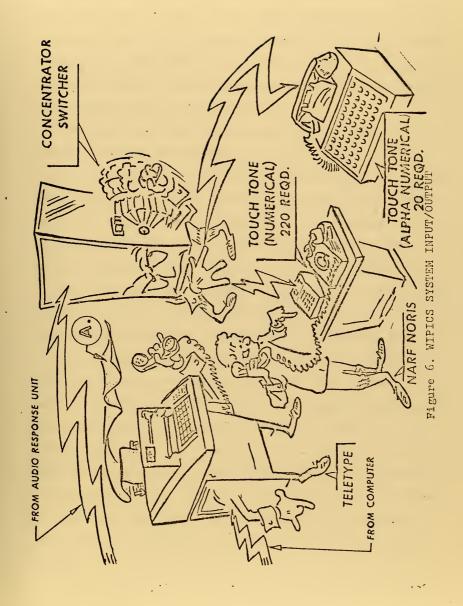
There exist two outlets for system information recovery. First is the audio-response program which answers inquiries over one of the 186 terminals. Secondly, the teletype-writer will print information as requested either on-line to simple inquiries or off-line to more complex needs.

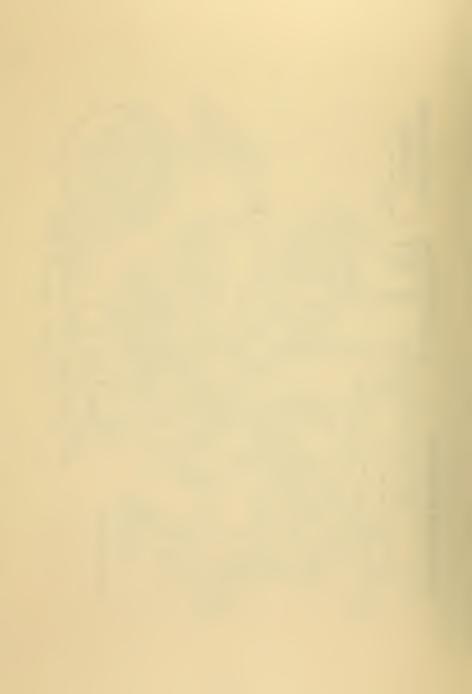
[Figure 6] These two outlets will provide item location,











its present and projected status and the relationship to all other items. Items in this case referring to individual engines or aircraft and any sub-assembly portions of either required for completion of production. Questions such as listed in Figure 7 can be answered at any time with clarity and without difficult interpolation.

How does the WIPICS system work in an operational sense? For each work-in-process item a location control record is established when the item first enters the system. This entry may be as the aircraft item rolls in the hanger or as the engine item is dismantled from an aircraft. The record contains such things as the control number, part number, register number, location data, and schedule data. As the item progresses through the maintenance schedule the computer is kept posted by each shop or work center as it arrives and as the work is completed. A necessary link to this system is the pre-programmed operations listing. Each operation to be performed is identified in terms of work to be done, material required and skilled labor needed for completion of the operation. When the scheduler in the assembly shop, for instance, is planning the next day's work he can touchtone a query [Figure 8]. The results can be received by audio response or printed readout, whichever is required. This real time information is invaluable, when trying to optimize utilization of men, material and money.



WHERE IS IT?

HOW LONG HAS IT BEEN THERE ?

WHEN IS IT DUE ?

WHY IS IT ON "HOLD" ?

WHAT IS ITS PRIORITY ?



WHAT PARTS ARE MISSING FOR THIS PARTICULAR

AIRCRAFT, WHAT IS THEIR SCHEDULE? WHERE ARE THEY ?

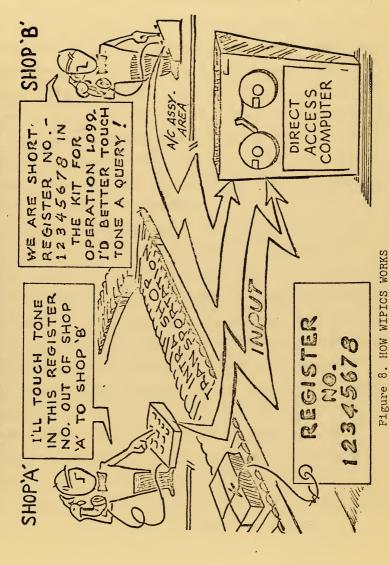
WHAT ORDERS DO WE HAVE IN WORK

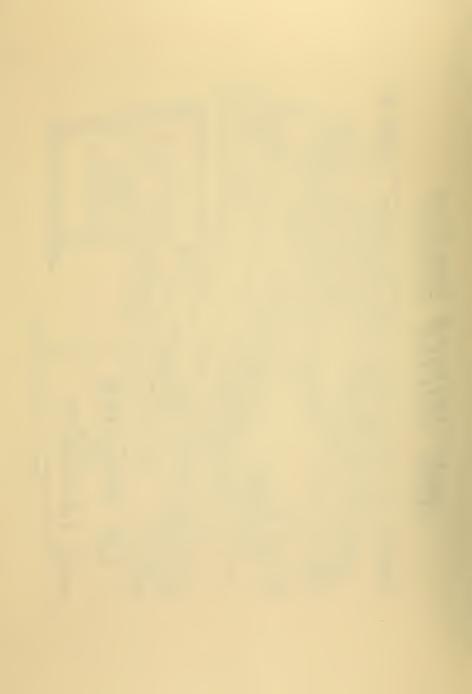
FOR THIS PART NO. ?

Figure 7. QUESTIONS ANSWERED BY WIPICS



sugal solding how





The production control staff is provided with real time data regarding condition, status and location of all work—in-process. For this work—in-process a listing can be obtained based on the part identification input or register number irrespective of the rework program under which the items are being processed. Data can also be retrieved by a family code. In order to avoid multiplicative errors, all input data is checked against previous files for correctness and a daily printout is made of all lost items and/or illogical inputs. Occasionally, auxiliary files and tables may be used for reference.

A transaction backup record is maintained and can be used for recall as needed. The use of this backup in quality assurance work alone is important. For example, if an aircraft, engine or component is inspected and found to be defective in a particular sub-assembly the backup record will list all effected products or unused assemblies. In addition, if this defect was caused by a human error the responsible shop personnel may be listed and retrained. The potential is there to expand WIPICS into other areas of the plant. Input of attendance and labor data could eliminate time clocks and make less work for disbursing clerks. Technical changes could be incorporated and continuously monitored and the all-automated storage areas would be easy to integrate with this new system.



II. THE PROBLEM

In recent years, Department of Defense policy has placed increasing emphasis on economy in the expenditure of its resources. The need for new equipment and new technology must be reviewed and analyzed. Consequently, the Navy is tasked with proving the need for new technology and for documenting its development from initial contract to final acceptance. The management of this development is critical and requires both qualitative and quantitative analysis.

The subject of OPNAV instruction 5231.1 is Automated
Data System (ADS), Development and is concerned with the
procedures for the management of these developments. An
enclosure to this instruction is the ADS development plan;
whose purpose is to provide a comprehensive, detailed
justification of ADS development, conversion or major
revision proposals. The plan is designed to answer these
fundamental questions: (1) Where are we? (2) Where do we
want to be? (3) What specific steps are we going to take?
(4) Who is responsible? (5) What resources are required?
(6) Is the trip worthwhile? These questions are addressed
in the economic analysis part of each new development plan.
The problem addressed in this paper is how does one prove
that the trip was in fact worthwhile?

Naval Operations, Chief of, OPNAV Instruction 5231.1, Subject: Automated Data Systems Development, Encl: (2), p. 1, 30 May 1972.



A. ECONOMIC ANALYSIS REQUIREMENTS

The economic analysis part of the development plan must present analytical justification of the proposed ADS development, conversion or major revision. WIPICS, since conception, has had such a plan and all milestone requirements have been met as WIPICS progressed to completion. The development plan requires a description of the before WIPICS system and statement about its objectives. The new system, WIPICS, is to be compared in terms of what assistance it provides toward satisfying the stated objectives of the old system.

The format for the economic analysis is carefully outlined into sections to give uniformity to different areas of analysis and to make the next higher command review easier. The first section is a brief synopsis. Secondly, the problem must be stated or, in this case, the opportunity for improvement must be described. This statement must be written in such a manner that continuous review is facilitated. The following is a list of sections in order, each is self-explanatory and must deal with the problem as stated in section 2. Section 1 - Synopsis, Section 2 - Problem Statement, Section 3 - Environment Description, Section 4 - Objectives, Section 5 - Assumptions and Constraints, Section 6 - Alternatives, Section 7 - Costs, Section 8 - Benefits, Section 9 - Comparison of Alternatives,



Section 10 - Sensitivity Test, Section 11 - Milestone Progress Report.²

The major objective of WIPICS, as stated in the Rohr proposal is to provide a tool for the NARFNI production control staff which will assist them in achieving reduced repair and overhaul cycles through improved parts location and status control. This objective clearly relates to the before WIPICS system and calls for a specific improvement. Inherent in the objectives of the original proposal was a subsequent reduction in labor cost and material dollar expenditures. In order to gain SECNAV approval, this plan had to estimate cost savings and system benefits which exceeded the existing systems potential for improvement. The improvement was predicted to be of sufficient value to offset the cost of the contract with Rohr Corporation.

B. COST SAVINGS AND SYSTEMS BENEFITS

This paper will be primarily concerned with the economic analysis of WIPICS in two of the eleven sections previously mentioned, cost savings and system benefits. The primary objective is to formulate a method of evaluating the new system improvements and performances versus the old systems performance.

In order to formulate a method of evaluation the entire NARF program came under scrutiny in an attempt to eliminate unimportant inputs as well as to better understand the

²<u>Ibid.</u>, pp. 3-12.



forecasted improvements documented in the initial analysis. To evaluate the success or failure of the new system a baseline had to be established from the data available. The data cover a period of nearly two years prior to the introduction of WIPICS and documents such items as repairs completed, labor costs, material costs, indirect labor costs and overhead expenses. It must be demonstrated that WIPICS either increased or decreased production output with the same budget. This paper will provide a method by which the success or failure of WIPICS can be documented.



III. GENERAL APPROACH TO THE PROGRAM

This section examines concepts that may be used to help identify and verify benefits derived from the implementation of a technological change in a production program. Understanding the useage and value of production functions, cost functions and certain mathematical techniques can make easier the evaluation of new systems.

A. PRODUCTION FUNCTIONS

The production of an output by using several inputs or resources can be modeled as a production function of the form,

(3-1)
$$Q = F(X_1, X_2, ..., X_n)$$

where Q is (homogeneous) output and X_i's are variable inputs to the function. The validity of a particular production function is determined by how closely the relationship between inputs and output approximates reality. Fo find the production function that best describes the relationship of a firm's output to their input requires an understanding of the firms objectives, operations, and actual input and output data.

The production function may be depicted by a table, a graph, or an equation which shows the maximum rate of output subject to a specified set of inputs. A table such as Table 3.1 is a form of production function Q = F(x)



OUTPUT	LABOR
(units)	(# men
16	1
33	2
50	3
60	4
70	5
75	6

Table 3.1 OUTPUT VERSUS LABOR INPUT

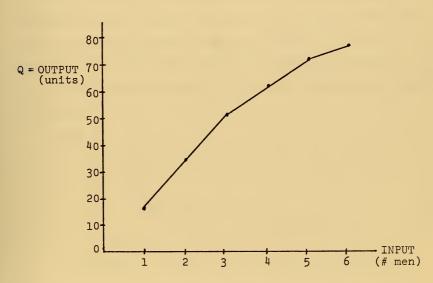


Figure 9. OUTPUT VERSUS SINGLE VARIABLE INPUT



indicating output achieved from a single input. This information could be represented on a graph, as in Figure 9.

From the table 3.1 and the graph in Figure 9, an equation could be derived that would represent the production function. This is a simple case since the function is dependent on only one variable.

A more interesting but more complicated case is where the production function is dependent on two variables. In order to extract information from a two input table or draw a graphical representation it is common to hold one variable constant or look at the combinations of variables that produce the same output. Figure 10 shows equal production points for different combinations of inputs. The two variable cases can give a series of constant production points which when smoothed and connected together are called isoquants.

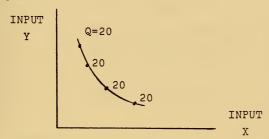


FIGURE 10. Constant Output From Two Variables



"An isoquant shows the various combinations of two inputs that result in an equal output for the firm." Given the production function, one can derive an isoquant pertaining to any level of output. The isoquant will be a representation of only efficient combinations of inputs. For example, if 2 units of output can be produced by 2 units of X_2 and 3 units of X_1 as well as by 2 units of X_2 and 4 units of X_1 , the latter will be ignored because it is obviously inefficient.

Production equations may be additive, multiplicative or any mathematical combination that represents the production of any output. The equations that follow are common examples of just such representations.

Q represents output, \boldsymbol{X}_1 and \boldsymbol{X}_2 inputs and a,b,α,β are all constant parameters.

- (1) LINEAR RELATIONSHIP (3-2) Q = ax₁ + bx₂
- (2) NON-LINEAR RELATIONSHIP (3-3) $Q = ax_1 + bx_2^2$ (3-4) $Q = ax_1^{\alpha} x_2^{\beta}$

Equation (3-4) is known as the Cobb-Douglas Function. This function is dependent on the values for α and β . The relationship between an input variable and output is linear

³Mansfield, Edwin, Microeconomics, New York: W. Norton Co., p. 133, 1970.



if the α and β power is equal to 1 when the other input is fixed. If not equal to 1, the relationship is non-linear. The parameters α and β represent the percentage increase of a unit of output when one unit of related input is added and all other variables remain the same. The significance of α and β when their sum is less than 1 is that the law of diminishing returns applies to the output results. The law has been documented in the "real world" many times over and simply states that, at a certain point, as equal increments of an input are added, the output results do not increase proportionally. This law assumes all other variables are held constant! Increasing returns require that $\alpha+\beta$ be greater than 1.

B. COST FUNCTIONS

Efficient production is that which utilizes the least input to produce the desired output. The least input will require the smallest outlay of constant dollars. In the case of multiple inputs the decision as to what mix to use for production is a function of the cost of those individual inputs. "To minimize the cost of producing a given output, a firm must combine inputs so that the marginal product of a dollar's worth of any one input is equal to the marginal product of a dollar's worth of any other input." In the case where the price of inputs are constant the most

⁴<u>Ibid.</u>, p. 184.



efficient point of production can be found where the cost line is tangent to the isoquant at the output desired.

(3-5) Cost =
$$C = 2X_1 + X_2$$

Equation (3-5) is a cost equation in which the price of x input is twice that of y input. Figure 11 illustrates how this cost function is to be used to find the optimum mix of inputs. The production output isoquant (A) and the cost curve (C) are tangent at point (B) and indicates the optimum mix of X_1 and X_2 .

C. LAGRANGE MULTIPLIER TECHNIQUE

"This method (the method of Lagrange Multipliers) can be used to find the maximum or minimum value of a non-linear objective function that is constrained by one or more equality constraints."

The minimization of a cost function subject to production function constraints can be solved by the Lagrange technique. For problems with four constraints or fewer, this technique is computationally acceptable. When more constraints are present, other methods such as sequential unconstrained minimization or geometric programming techniques may be used. 6

⁵Plane, Donald R. and Kochenberger, Gary A., <u>Operations</u>
Research for <u>Managerial Decisions</u>, Homewood, Illinois:
R. D. Irwin, Inc., p. 282, 1972

^{6&}lt;u>Ibid.</u>, pp. 284-291.



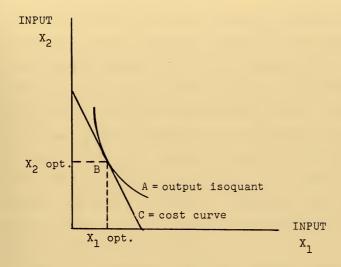


Figure 11. PRODUCTION MIX AND MINIMUM COST

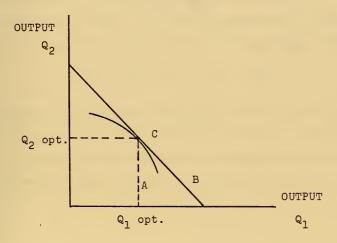


Figure 12. PRODUCTION TRANSFORMATION CURVE, VALUE LINE, AND OPTIMUM MIX



D. PRODUCTION POSSIBILITY CURVES

The production possibility curve is a locus of points showing the maximum attainable output of one commodity for every possible output of the other commodity while keeping a fixed resource base. Another name for production possibility curve is product transformation curve. Both refer to the same graphic description of production constrained by either dollars in a budget or resources available. The curve shows the relationship between two or more outputs subject to a single constraint. Generally, the trade-off between production of one good and another is not favorable and gives diminishing returns.

With a set constraint such as a budget, the only question remaining is at what combination of production is the value gained the greatest. The value gained may be found by using a value function that relates the value of two outputs.

Value Function =
$$V = Q_1 + Q_2$$

Figure 12 depicts how this value function may be used to find the optimum mix of outputs \mathbf{Q}_1 and \mathbf{Q}_2 . The optimum mix, \mathbf{Q}_{1c} and \mathbf{Q}_{2c} , is at the point of tangency between the transformation curve, A, and the value curve, B.



IV. PROBLEM SOLUTION

This section will look at the budget at NARFNI in non-quantitative terms. It will be shown how this budget can be transformed into a product transformation curve in terms of NARFNI output. This approach is possible by using Cobb-Douglas production functions and the Lagrange multiplier technique. The output is in terms of production load norms which bears a direct relationship to the repairing of aircraft, engines, and components. This repair process will be referred to as production. The relationship between input resources, prices, and norms is found mathematically and then substituted into the budget equation. The budgets product transformation curve may be compared to other budget curves. First the data provided must be analyzed.

A. DATA ANALYSIS

The NARFNI production department supplied data from an observation period of approximately 19 months. Data used from this period covered only work inducted after January 1970 which was completed by approximately September 1971. The data as listed in Appendix A contains information about the work done on individual jobs. In order to look at the work performed daily and the cost of that work each day, an assumption was made that each job expended resources

⁷ Naval Air Systems Command, NAVAIR INSTRUCTION 5220.2, Subject: Production Performance Report for Overhaul and Repair Departments, 12 October 1966. [Production Load Norms: Negotiated production expressed in terms of direct hours, determined at the quarterly Fleet Support Conference.]

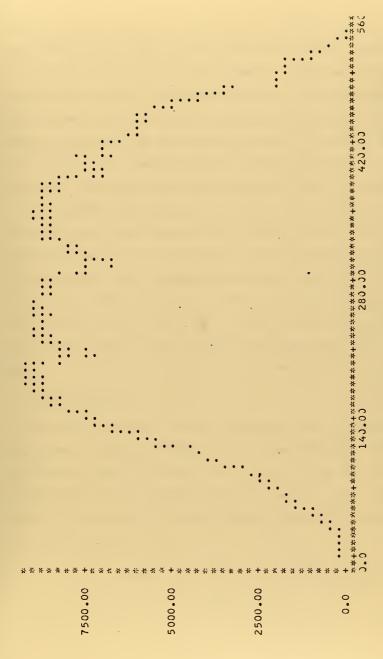


uniformly over the days in shop. The number of days a job was in shop was figured from the difference between induction date and completion date. This number considered holidays or week-ends as regular days, so the days that actually were used to work on the job is unknown. For instance, if an engine was in NARF ten days for repair, each of the ten days would be credited with 10% of the labor man-hours, labor costs, material costs and norm.

Appendix B contains a program to pro-rate and graph the data over the observation period. The graphs show a cyclic effect to be present in the data with a rise and a fall approximately every 90 days. Little work was recorded on the first day and the last day. In the beginning of the period, some work was being done by the NARF on jobs in process before observation day one. At the end of our sample some jobs have been worked on but have not been completed. In both cases the data on the work are unavailable. Consequently, the observation period had to be shortened to eliminate the "start up" and "shut down" influence on the continuous production model [Figure 13].

The program listed in Appendix C was used to graph and analyze the empirical distribution of the number of jobs in NARF over the observation period. The objective of this program was to find a shortened period of observation with 95% confidence that "start up" and "shut down" influence had been eliminated. Each job was put in order by the





GRAPH OF PRORATED NORM (Y AXIS) VS DAYS (X AXIS) FOR AIRCRAFT Figure 13. "START UP" AND "SHUT DOWN" EFFECT FROM DATA



number of days required to complete work. The first job on this ordered list took the fewest days for repair. When 95% of the jobs were ordered the number of days required to complete jobs at that point was recorded. The average number of days in shop, standard deviation, minimum and maximum number of days and the 95th percentile are all outputs from this program. An example of that output is depicted in Figure 14. The 95th percentile was used instead of trying to fit a known distribution to the date. The recorded number of days was then deducted from each end of the original observation period. The aircraft observations were cut by 122 on each end. The engines observations were cut by 47 on each end. The pro-rated data over the shortened observation period is the data used to determine the allocation of man-hours, labor costs, material costs and norm over time.

B. BUDGET EQUATION

The budget equation has two different types of variables, quantities of resources and price of those resources. The resources are labor, material and pipeline. They are used in the production of repaired aircraft, engines, and components. The budget is a dollar value allocated to production over a period of time.

(4-1) BUDGET =
$$\mathbb{B} = \mathbb{P}^{\mathbb{L}}(\mathbb{L}_1 + \mathbb{L}_2 + \mathbb{L}_3) + \mathbb{P}^{\mathbb{M}}(\mathbb{M}_1 + \mathbb{M}_2 + \mathbb{M}_3) + \mathbb{P}^{\mathbb{P}}(\mathbb{P}_1 + \mathbb{P}_2 + \mathbb{P}_3)$$



		* * * * * * * * * * * * * * * * * * *
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GRAPH OF NUMBER OF DAYS EACH JOB IS IN SHOP (AIRCRAFT) MEAN= 83.58 STD DEV= 34.39 MIN NO. CF DAYS I MAX NO. OF DAYS I 951H PERCENTILE O

SHOP= 28.0 SHOP=256.0 ABOVE DISTRIBUTION = 122 SHOP FOR AIRCRAFT ZZU. Figure 14. DISTRIBUTION OF DAYS IN



1. Resources Inputs and Subscripts

The labor input is measured by man-hours. Direct labor is that number of man-hours which can be accurately, feasibly or economically identified to the benefitting end-product. The material input is measured in dollars. The direct material costs are those which can be accurately, feasibly or economically identified to the benefitting end-product. The pipeline input is measured in job-days. Pipeline is the number of jobs present in shop. Each job in shop one day is equal to one job-day. For any specified period of time the number of job-days in shop can be calculated.

- a. Aircraft production inputs are subscripted with the number 1.
 - L_1 = man-hours of direct labor on aircraft
 - M_{γ} = direct material costs used in aircraft
 - P_1 = number of aircraft days in shop
- b. Engine production inputs are subscripted with the number 2.
 - L₂ = man-hours of direct labor on engines
 - M_{2} = direct material costs used in engines
 - P_2 = number of engine days in shop
- c. Component production inputs are subscripted with the number 3.
 - L_2 = man-hours of direct labor on components
 - M_2 = direct material costs used in components
 - P_3 = number of component days in shop



2. Prices

The price of labor is an average price of labor per hour. This price is found by aggregating all labor man-hours and dividing by the total labor costs. The price of labor in the production of aircraft, engines and components is the same. The price of material is equal to one because the resource input is in dollars expended on direct material. The price of pipeline is an average price of the job-day value. This is the price the Navy pays for not having the repairs completed. A civilian example would be the cost of "loaner" cars to an automobile dealer. The quicker the repair is completed, the less the cost. This price is found by dividing the replacement cost of the repaired job by the number of projected useful days after the repair is completed.

 P^{L} = average price of labor per hour

 $P^{M} = 1$

P^P = average price of pipeline per day

C. PRODUCTION FUNCTIONS

The production functions are Cobb-Douglas models of the production process at NARFNI. The function consists of constant parameters determined by historical data and previously defined labor, material and pipeline resources. The output of the functions is norm.



(4-2)
$$Q_1 = a_1 L_1^{\alpha_1} M_1^{\beta_1} P_1^{\gamma_1}$$

$$(4-3)$$
 $Q_2 = a_2 L_2^{\alpha_2} M_2^{\beta_2} P_2^{\gamma_2}$

$$(4-4)$$
 $Q_3 = a_3 L_3^{\alpha 3} M_3^{\beta 3} P_3^{\gamma 3}$

1. Ouput

Norm, the output of the production functions, is a flexible measurement of production and is in terms of manhours required to repair one job. This number relates to the previous norm and is constantly being updated.

 Q_{γ} = Aircraft norm

Q₂ = Engine norm

 Q_3 = Component norm

2. Inputs

The same resources in the budget equation, L, M, and P are used in the production function. As previously defined, the subscripts, 1, 2, and 3 identify the resource used for repair of aircraft, engines and components respectively.

3. Parameters

The parameters use the same subscript scheme for reference to a particular output. The numerical value for the parameters a, α , β , γ is found by investigating the production data as presented in the production performance



reports. The estimation of these values is being done by concurrent thesis work using econometric techniques. The powers associated with each of the resources labor, material and pipeline are α , β , γ respectively. These resource parameters will generally be less than one which requires that the marginal product of an input decreases with increases in its utilization. ⁸ For an example, $L_1^{\alpha 1}$ means that if one percent of L_1 were added to production, that α percent increase in Q_1 would be gained.

D. LAGRANGE TECHNIQUE AND SOLUTION

From the problem:

Min B =
$$P^{L}(L_1+L_2+L_3)+P^{M}(M_1+M_2+M_3)+P^{P}(P_1+P_2+P_3)$$

S. T. $Q_1 = a_1L_1^{\alpha_1} M_1^{\beta_1} P_1^{\gamma_1}$
 $Q_2 = a_2L_2^{\alpha_2} M_2^{\beta_2} P_2^{\gamma_2}$
 $Q_3 = a_3L_3^{\alpha_3} M_3^{\beta_3} P_3^{\gamma_3}$

The Lagrange Equation:

$$(4-5) \qquad (L,M,P,\lambda) = P^{L}(L_{1}+L_{2}+L_{3}) + P^{M}(M_{1}+M_{2}+M_{3})$$

$$+ P^{P}(P_{1}+P_{2}+P_{3}) - \lambda_{1}(Q_{1}-$$

⁸ Mansfield, p. 141.



Appendix E has six steps which were taken to use the Lagrange relationships to restate the budget equation in terms of output. Step five of Appendix E presents the outputs Q_1 , Q_2 and Q_3 in terms of each resource raised to a power multiplied by a new constant.

$$Q_1 = AL_1^u = BM_1^u = CP_1^u$$

$$Q_2 = DL_2^V = EM_2^V = FP_2^V$$

$$Q_3 = GL_3^W = HM_3^W = JP_3^W$$

The budget equation (4-1) can be rewritten in terms of output as is done in step 6 of Appendix E. The result of the substitution is a non-linear relationship between \mathbf{Q}_1 , \mathbf{Q}_2 and \mathbf{Q}_3 .

$$(4-6) \quad \mathbb{B} = K_1(Q_1)^{\frac{1}{u}} + K_2(Q_2)^{\frac{1}{v}} + K_3(Q_3)^{\frac{1}{w}}$$

where K_1 , K_2 and K_3 are constants.

If u, v and w were all equal to one, the relationship between outputs would be linear.

E. GRAPHICAL ANALYSIS

An iso-budget surface in three dimensional output space would represent the production possibility surface. However,



three dimensional graphs are difficult to draw as well as interpret. Consequently, only two outputs will be used, and the third will be set equal to zero in this sample analysis. The results can be extended to three dimensions. The budget in terms of aircraft and engine output only is:

$$(4-7) \quad \mathcal{B} = K_1(Q_1)^{\frac{1}{u}} + K_2(Q_2)^{\frac{1}{v}}, Q_3 = 0$$

a non-linear product transformation equation. The slope of the product transformation curve is dependent on the values of u and v. Appendix F derives the second derivative in order to find whether the curve is convex or concave to the origin. The shape of the transformation curve will indicate increasing, constant or decreasing returns to scale [Figure 15]. The relationship between outputs is expected to be explained by the law of diminishing returns. This relationship is depicted by a concave product transformation curve.

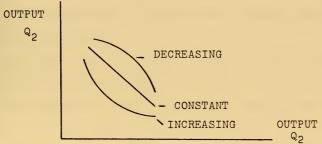


FIGURE 15. Product Transformation Curves, Returns to Scale



Engine output in terms of the budget and aircraft output is eq. (4-8). The second derivative of this expression

$$(4-8) \quad Q_2 = (\frac{B}{K_2} - \frac{K_1}{K_2}(Q_1)^{\frac{1}{u}})^{v}$$

with respect to Q_1 , aircraft output, will determine convexity or concavity. A concave curve will have a positive second derivative and the convex curve will have a negative second derivative. By inspection the sign_1 of Q_2 " is controlled by the expression $B(u-1)+K_1(v-u)Q_1^{\overline{u}}$. Substitute Eq. (4-7) for B and the sign of Q_2 " may be found by analyzing the expression

$$K_1Q_1(v-1) + K_2Q_2(u-1)$$
.

If u and v are both > 1 the curve is convex.

If u and v are both < 1 the curve is concave.

If u and v are both = 1 the change in slope is zero indicating linearity.

If the values of u and v are > 1 and < 1 respectively, the expression must be further evaluated.

The appropriate curve in Figure 15 can be drawn from Eq. (4-7). It should be interpreted only over the range of outputs that have been observed. In addition, the production data reveals that output of aircraft, engines and



components does not vary greatly from quarter to quarter. The three curves in Figure 15 are examples of possible product transformation curves. If the relationships derived and the data utilized are valid the product transformation curve should show all optimum combinations of production possible with a set budget.

A comparison may be made between before WIPICS and after WIPICS transformation curves [Figure 16]. These curves would have to represent the same total budget dollars and the prices of the resources and inputs would have to be expressed in constant dollars. Figure 17 depicts the situation where the better or worse evaluation question depends on the production output ranges. For example, better production would be achieved after WIPICS, only if $Q_1 \geq Q_{1a}$ and $Q_2 \leq Q_{2a}$. If while holding one output constant the new transformation curve provided more production of the other output the resultant savings would be that numerical increase in production multiplied by the value of output. Further analysis would be required to determine the exact dollar savings or loss after the purchase cost of WIPICS was deducted.

F. VALIDATION

The validation of the Cobb-Douglas production models used in this paper will be performed by a comparison of results with a linear economic model and the actual production results as documented at NARFNI. The Cobb-Douglas



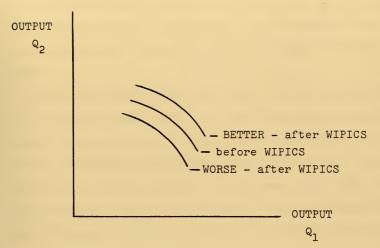


Figure 16. TRANSFORMATION CURVES, CLEAR RESULTS

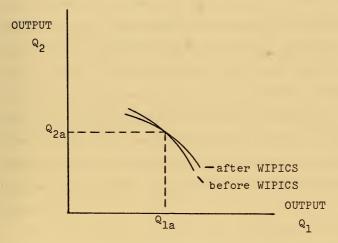


Figure 17. TRANSFORMATION CURVE, MIXED RESULTS



models are derived from the observed production data taken from the Production Performance Reports (PPR) for Fiscal Years 1970 and 1971. (Appendix A). Consequently, the comparison should be made only over the ranges of output observed.

Linear programming techniques may be used to formulate three linear economic models for the production programs at NARFNI. The objective functions will be the budget function and the constraints can be any number of documented relationships in the repair program. For instance, the engine production program consists of different repairs on different types of engines as well as different repairs on the same type engines. How these repairs are aggregated, if at all, will determine the number of constraints in the general linear program. The simplex algorithm must be used to solve the multi-variable linear program. Since the data used in the L.P. comes directly from the PPR and the data covers a two-year period of time, the linear economic models will closely model the production programs. All three linear models can be examined in detail by using sample production requirements and conducting parametric studies to determine the sensitivity of the solutions obtained.

The desirability of using the Cobb-Douglas production function over the general linear programming model lies in the simplicity of utilization of the former. Once the aggregation of the data has been done, the regression



coefficients in the Cobb-Douglas function are computed by applying the principle of least squares to the logarithmic values of the variables. This linear transformation is made because calculations of the coefficients are easier and statistical theory is primarily developed under the assumption of linearity.

A comparison of the Cobb-Douglas production model, and the linear programming model can be made by comparing their product transformation curves to the actual production output. Cobb-Douglas and linear programming models should depict similar curves for equal budgets. For a particular budget the actual production will be a unique point as represented in figure 18. The area ABCD represents the area of possible output combinations after plotting historical ranges of each output. With the budget held at the same constant dollar level the comparison between models and

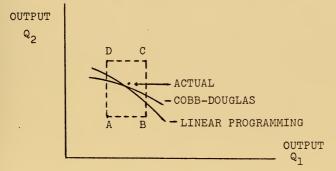


Figure 18. Validation of Transformation Curves

⁹Chou, Ya-Lun, Statistical Analysis, New York: Holt, Rinehart and Winston, Inc., p. 657, 1969.



actual data is easier. The model curves can be compared for goodness of fit to a series of actual production points. The decision as to how well either curve should represent the data must be determined.

Another way to evaluate the accuracy of the models is in terms of constant outputs. This method will compare the budget for actual output with the budgets required for the models to match that output. The difference between the budget costs will give a measure of how close the models are to representing actual production. Many different actual production combinations can be considered and in each case the dollar difference in the required budgets can be measured. This form of comparison still requires a decision to be made about how close the budgets must be to the actual budget before the production models are accepted for use in the analysis.



V. CONCLUSIONS AND FUTURE STUDY

A. CONCLUSIONS

The initial efforts of this research paper were directed toward the analysis of data from the Production Performance Reports supplied by NARFNI. It was shown from the data that a problem existed with a "start up" and "shut down" effect. A method of eliminating that problem resulted in the pro-rating of the data in order to find an unaffected shortened period of observation. Once the observation period was determined, efforts were directed toward the determination of production functions which adequately represented the production process at NARFNI. Research revealed that the Cobb-Douglas production functions could be used to represent the entire NARFNI production process. Cobb-Douglas functions require coefficients of production which must be derived from the observed data. Determination of those coefficients can be accomplished by using econometric techniques. A linear programming model was proposed as a useful validation tool in addition to the comparison with actual data. The problem of establishing the validity of the models proposed is mentioned and possible approaches to finding solution are offered.

The solution to the problem as stated in this paper lies in the use of the product transformation curves. These curves can represent production in such a way that expensive,



time consuming measurements of many variables are not required and detailed relationships between shops and programs need not be analyzed. It has been shown that the production functions are sufficiently detailed to permit a mathematical look at the trade-offs between inputs. For example, the trade-off between labor, man-hours, and pipeline costs can be found given some level of output. It has been shown that the ultimate evaluation of WIPICS, before and after, can be made by comparison of the outputs from the constant dollar budget or the comparison of the budgets from the constant output.

B. FUTURE STUDY

The prototype WIPICS contract is not complete until the end of December 1972. Data from the period of time while WIPICS was being installed and personnel were learning the system should be considered as biased data. The best measure of the system would be to compare the 1970 and 1971 fiscal year production data against the data from 1974 and 1975. Even with the elimination of the phase in time, numerous factors could effect the data during the two different periods completely independent from WIPICS. NARF management and policy decisions on the national level as well as on the local level would have an effect on production regardless of the incorporation of WIPICS. The ultimate relevance of external factors would have to be decided and allowed for in the final WIPICS cost-benefit analysis.



An area that must be investigated is the actual dollar savings or loss from WIPICS. This dollar sum would have to consider the costs associated with new equipment, training, computer rental time and qualified technical personnel.

Just how the exact cost can be formulated is no easy task and is made even more difficult by the changing prices of the resources and the ever-present inflationary losses.

1. Sensitivity Analysis

The model as developed in this paper can be easily modified by changing prices of inputs or quantities of resources. Also, if the budget were increased or decreased the changes in the product transformation curves could be studied. These adjustments to the model may provide insight to future production benefits or problems.

2. Cyclic Phenomenon

The graphs in Figure 13 and 14 display a cycle of production in the NARF data. An explanation of this may have a definite bearing on the pro-rated data and the subsequent use of that data in the production functions. The budget is reviewed quarterly and so is production planning. Possibly these events have an effect on the work-load in the NARF.

3. Computer Use

The programs in Appendix B and C could be combined in one program. This would give a graphic display of the basic data as well as a graph of the pro-rated data over



the shortened observation period. A comprehensive program could include the minimization of the budget subject to the production function constraints. The parameters required in the production functions could be solved by a regression model in the same program. The linear programming problem could be solved simultaneously with the minimization problem as presented in this paper. The results could then be viewed as a single computer output.

4. Different Models

Two other models of production should be considered. The constant elasticity substitution (CES) production function and the stochastic production function. The latter, a version of the Cobb-Douglas model, simulates uncertainty in production expectations. The stochastic model would have the advantage of being able to determine optimal factor inputs on the basis of an expected output constraint.



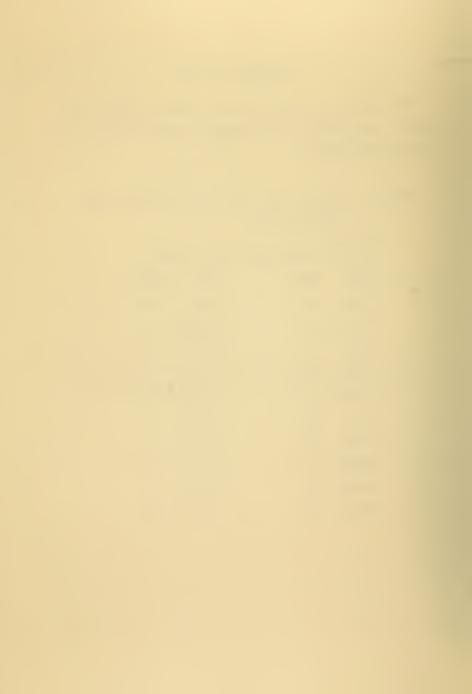
APPENDIX A. DATA

This appendix contains the raw data as received from NARFNI. The columns of the computer listing refer to the following information:

Column 1 - Special two digit thesis code for type $$\mbox{\sc A/C}$$ and engine.

Aircraft Type-Model-Series Codes

A/C	Code	A/C	Code
C-2A	10	SH-3A	33
E-2A/B	11	SH-3A/G	34
F-4J	21	SH-3D	35
F-4B	22	CH-46A	41
F-4G/B	23	CH-46D	42
F-8J	25	CH-46F	43
F-8H	26	UH-46A	44
RF-8G	27	UH-46D	45
CH-3B	31	CH-53A	48
RH-3A	32	CH-53D	49



Engine	Type-I	Model-	Series	Codes
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ENGINE	CODE
J57-P-4A/22	71
J57-P-10	72
J57-P-20A	73
J57-P-22	74
J57-P-420	75
J79-GE-8B	91
J79-GE-8B/C	92
J79-GE-8B(RDTE)	93
J79-GE-8B/C(RDTE)	94
J79-GE-10	95
T58-GE-1(A/F)	51
T58-GE-3(A/F)	53
T58-GE-5(A/F)	55
T56-A-8P	56
T58-GE-5(C/G)	57
T58-GE-8F	81
T58-GE-8B	82
T58-GE-8B/F	83
T58-GE-8B(C/G)	84
T58-GE-8B(HH2)	85
T58-GE-8B/F(CONV)	86
T58-GE-8B/F(RES)	87
T58-GE-10	89



T64-GE-3(A/F)	63
T64-GE-6B	65
T64-GE-6B(PAMN)	66
T64-GE-7(A/F)	67
T64-GE-413	69

Column 2 - Aircraft Bureau Number and Engine ID Number

Aircraft Categories and Cycles Numbers

A/C CATEGORY	FIRST DIGIT	SECOND DIGIT
Overhaul	1	Cycle Number
PAR	2	11
PAR/CONV	3	11
PAR/MOD	4	11
PAR/MOD/REP	5	11
PAR/REP	6	11
PAR/SEA/53	· 7	n .
PAR/CONUS/53	8	ıı .
New Additions	9	11

Engine Category

CATEGORY	CODE
Overhaul	01
Overhaul Conv.	02
PAR/REP	03



SUPPLY/REP 04

SUPPLY/REP/CONV 05

Column 4 - Induction Julian Date

Column 5 - Completion Julian Date

Column 6 - Norm

Column 7 - AFC Man-hours (A/C only)

Column 8 - Direct Man-hours

Column 9 - Direct Labor Cost

Column 10- Direct Material Cost

Column 11- Overhead Costs Applied

Column 12- NIF Hourly Rate



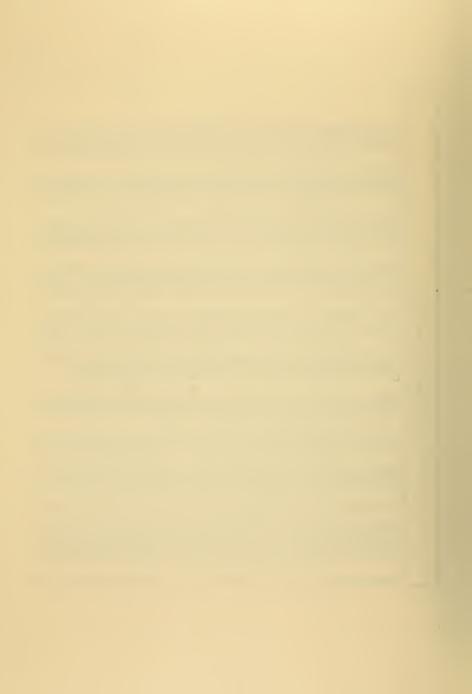
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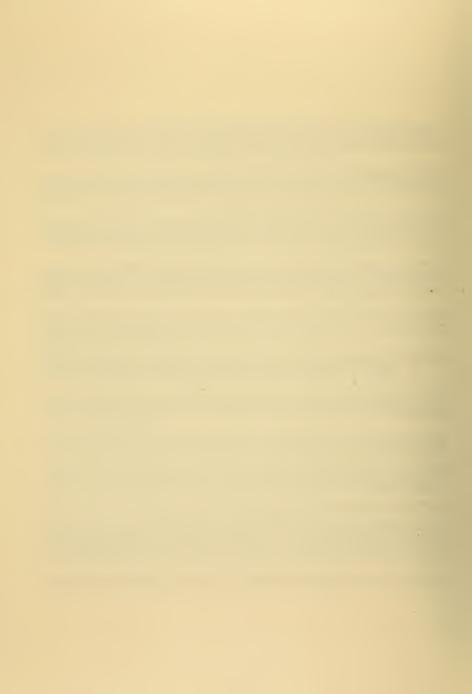
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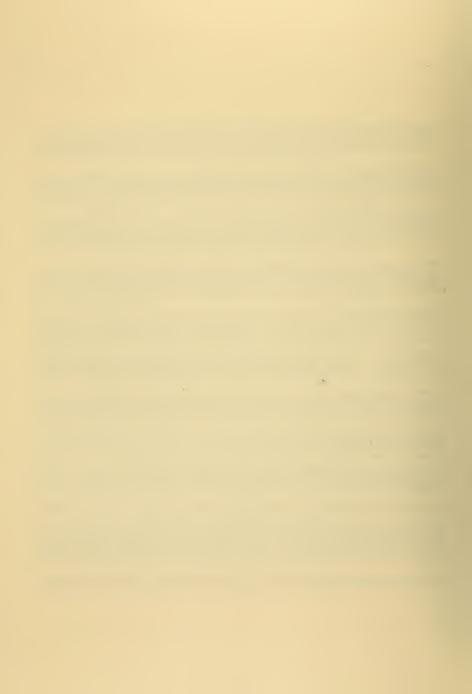
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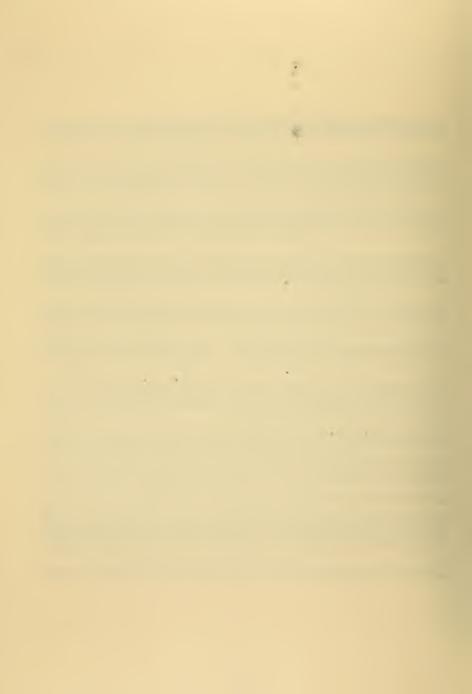
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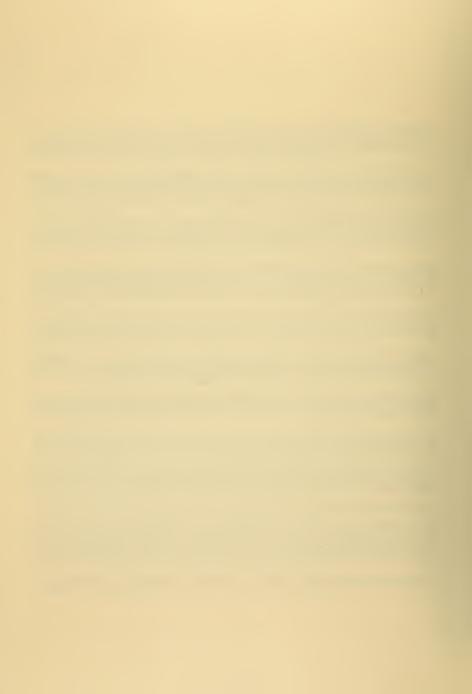




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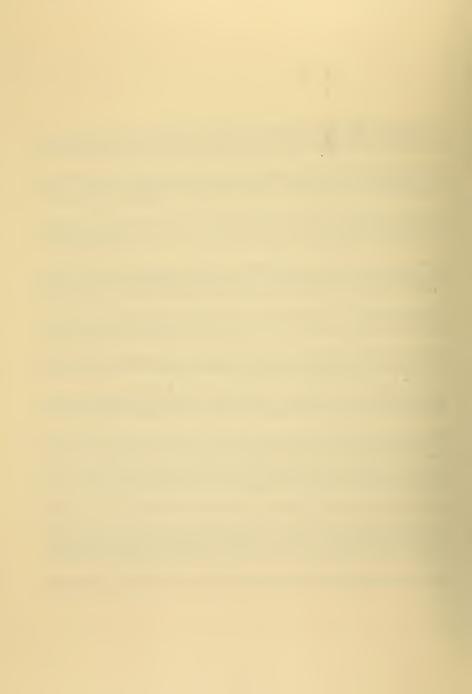


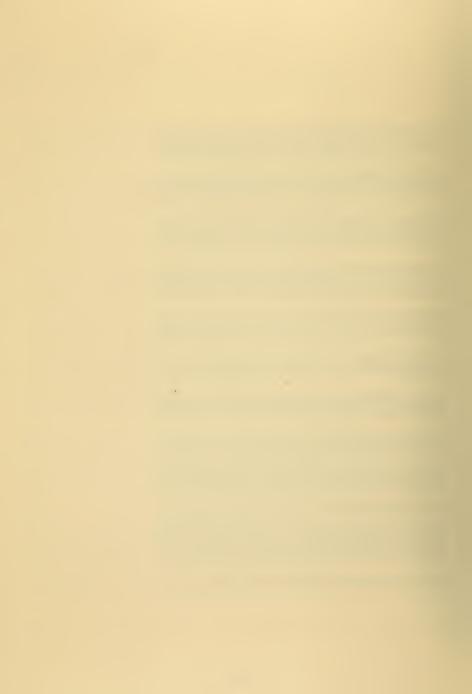
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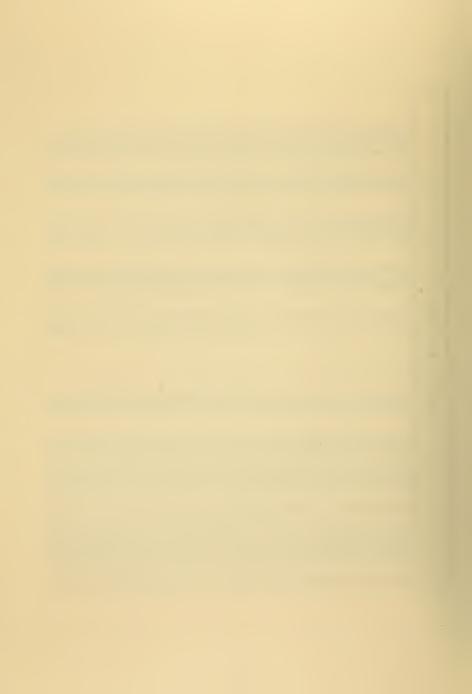
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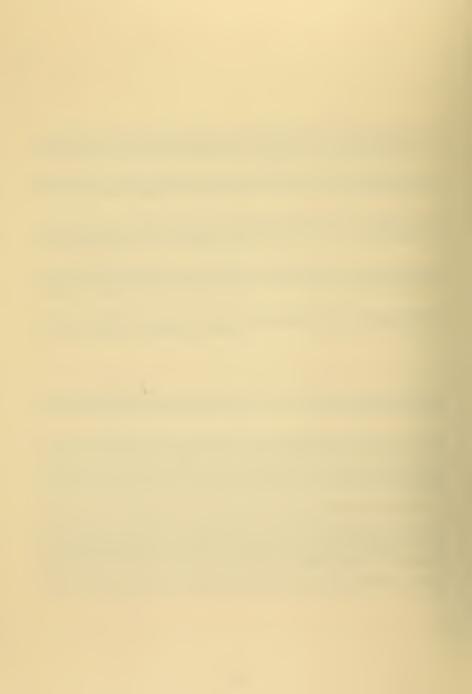




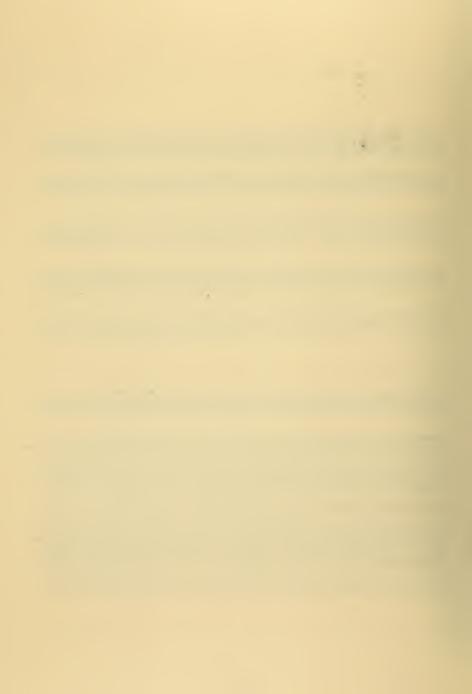
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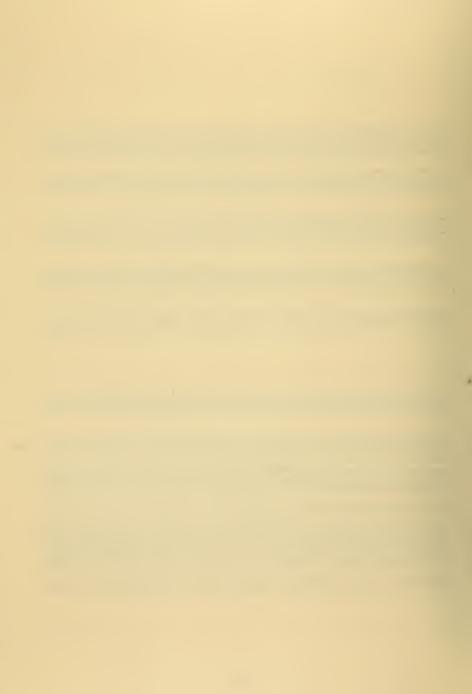


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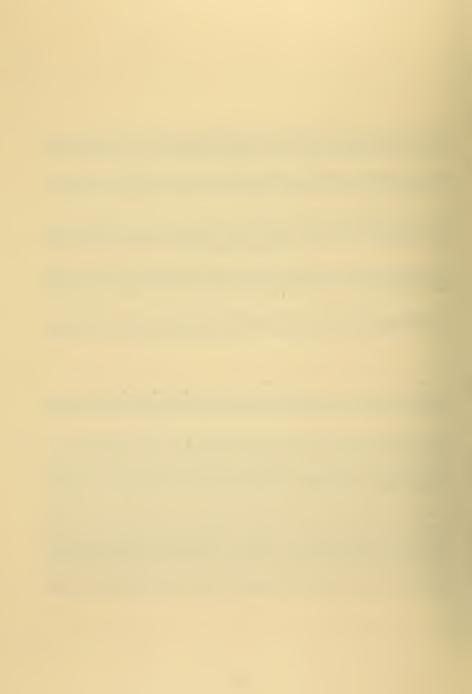


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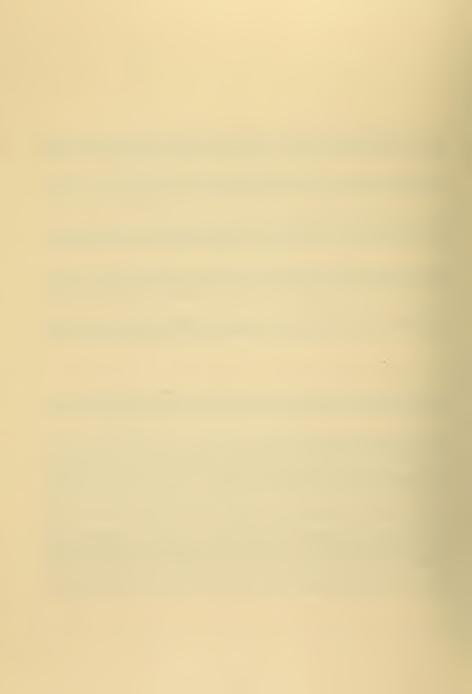


| Company | Comp

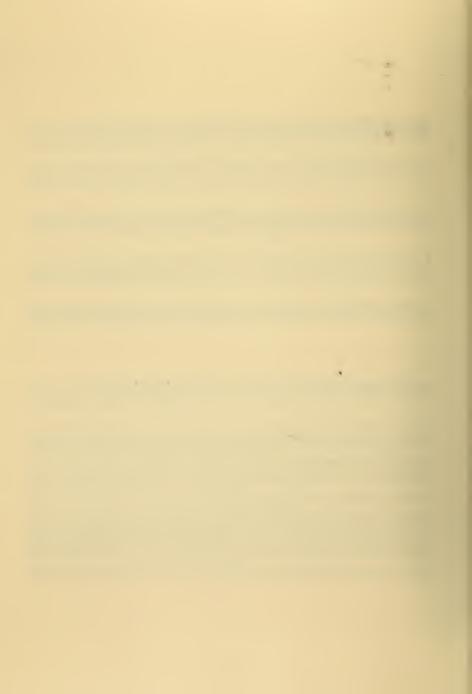


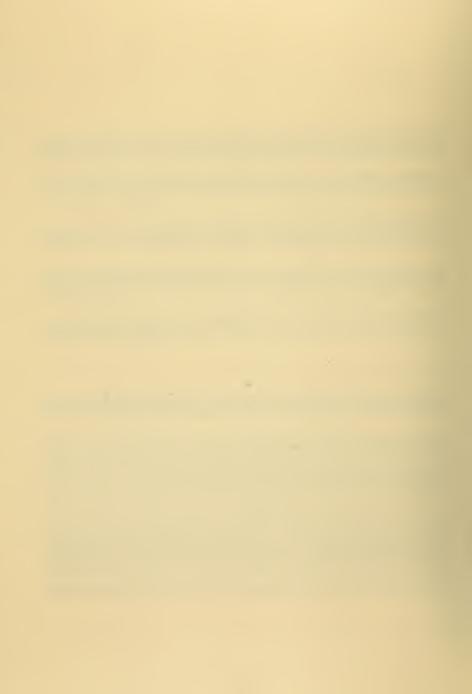
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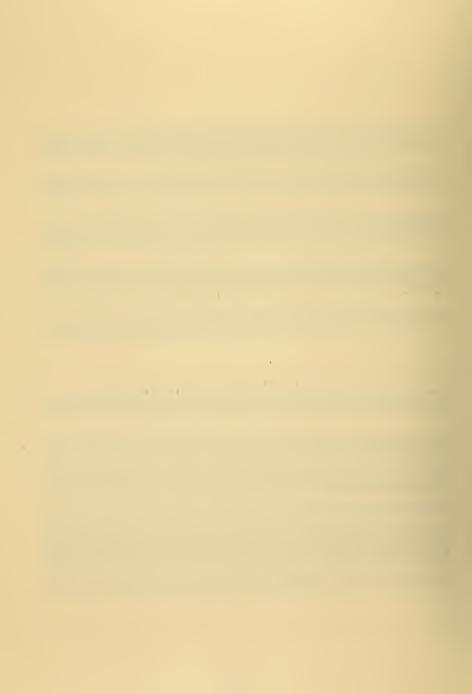
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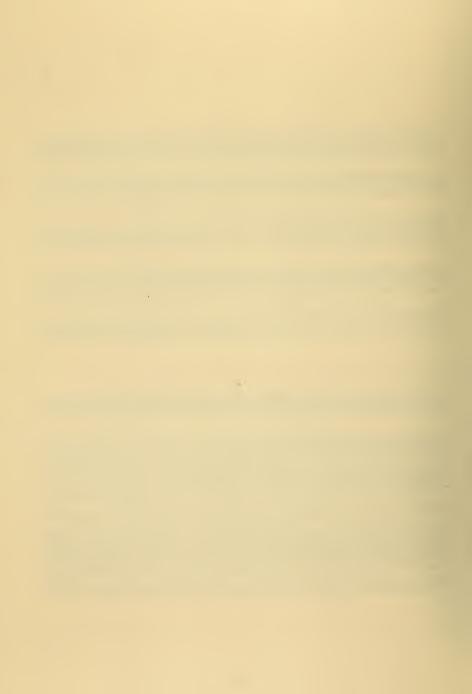


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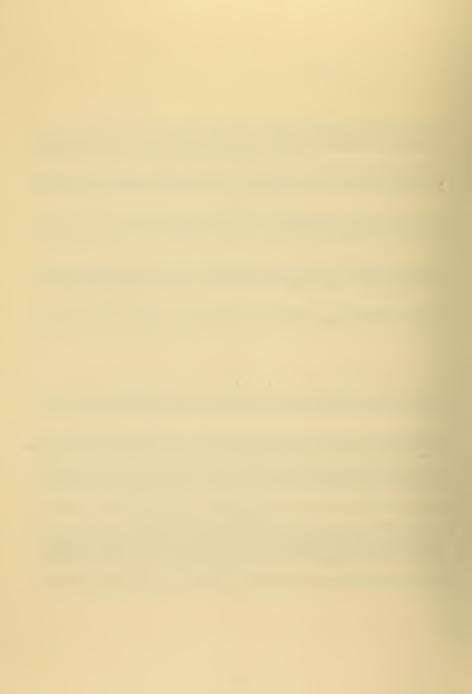
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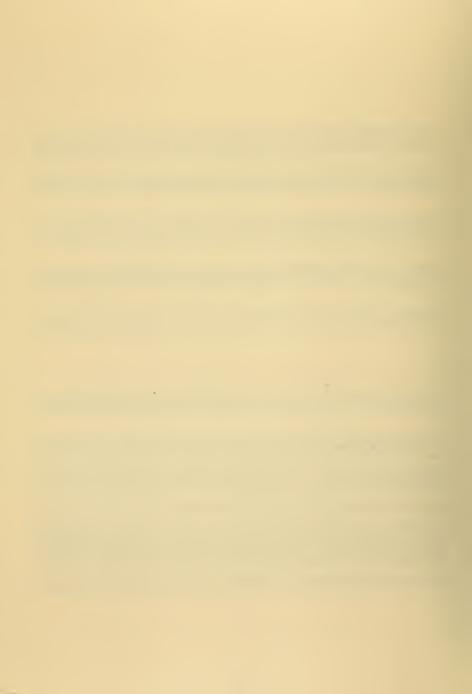
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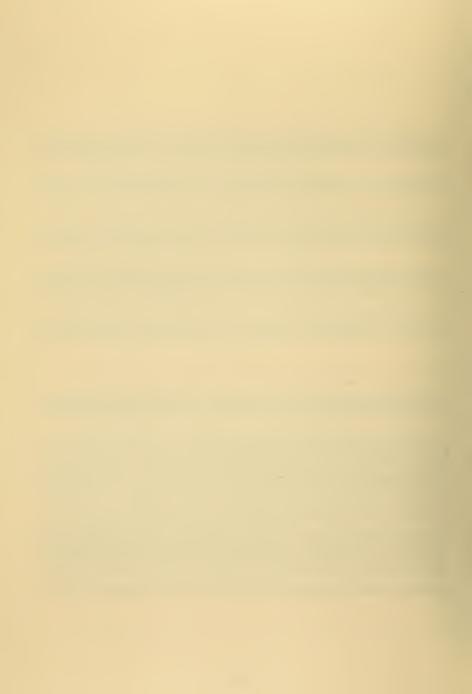
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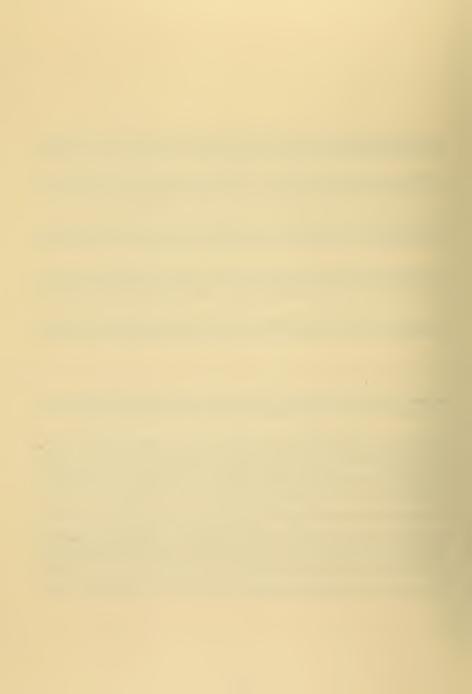
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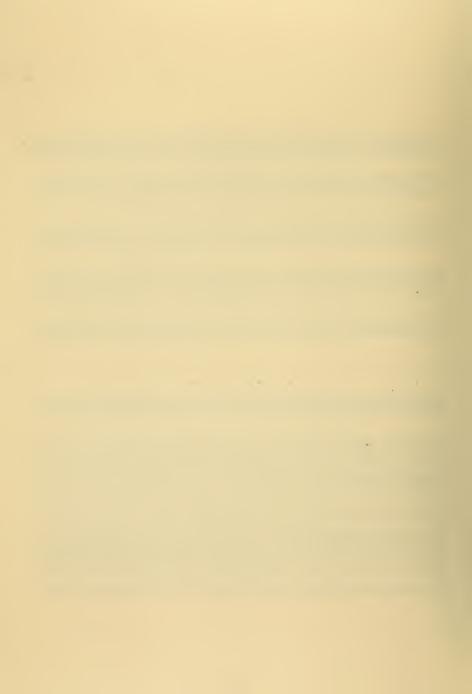




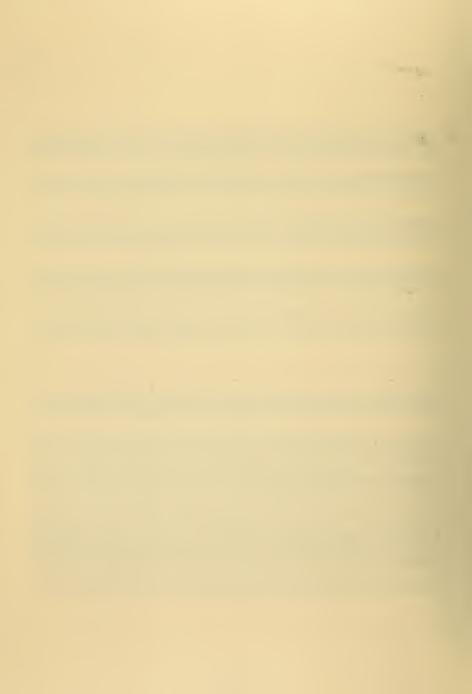
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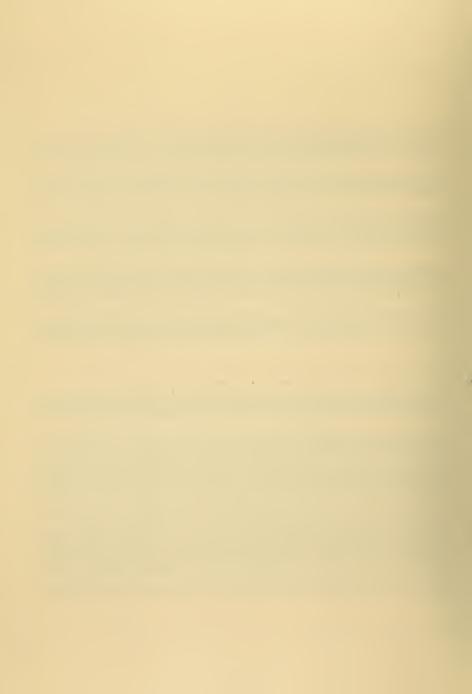
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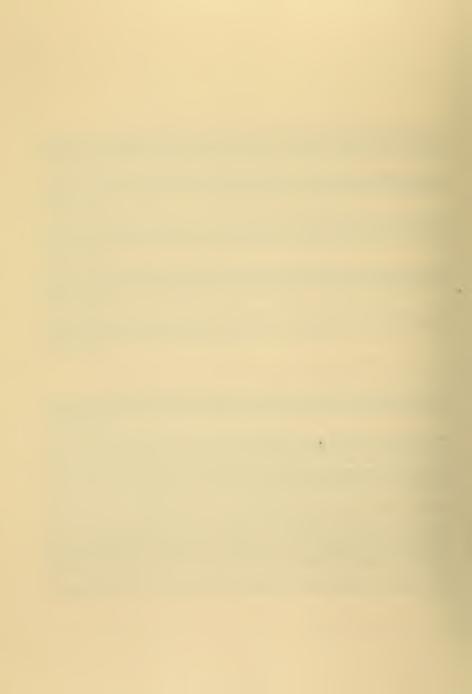
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| CANDALIVAL | CAN 4+0m4+0m404+0m40400m404mm404m4mm4mm404m444mm4 $\frac{1}{2} \frac{1}{2} \frac{1}$ σ



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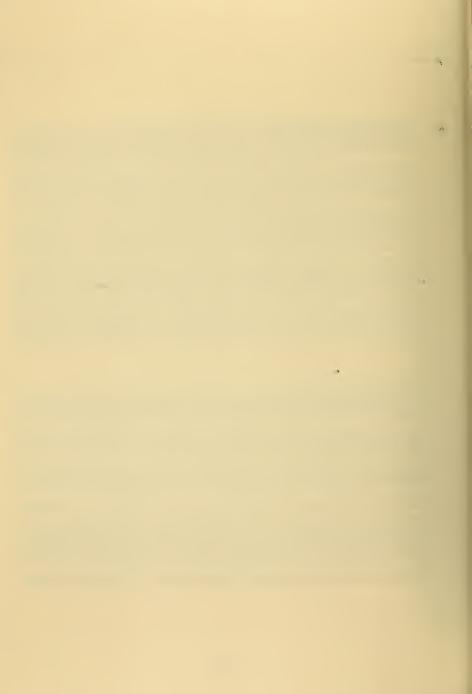


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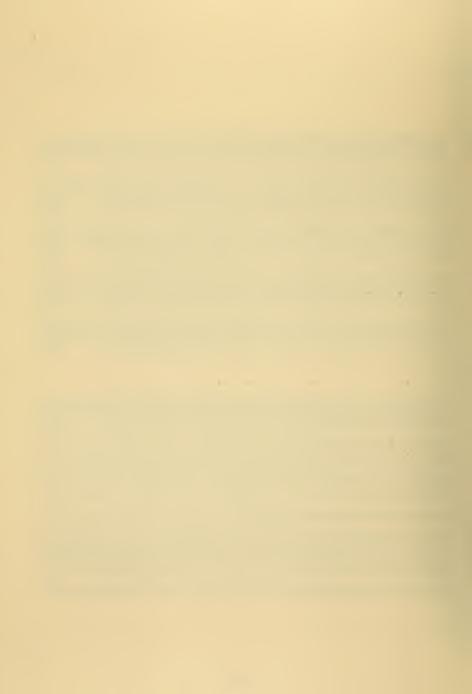
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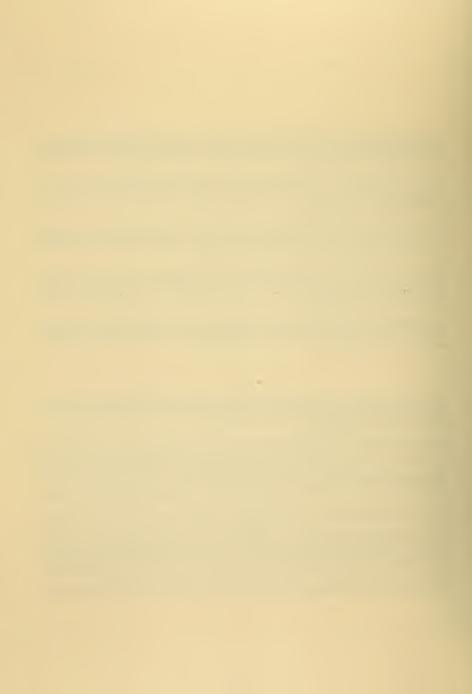


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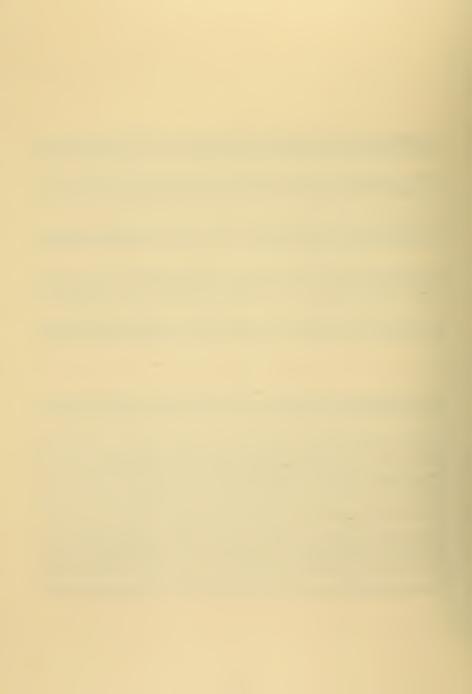


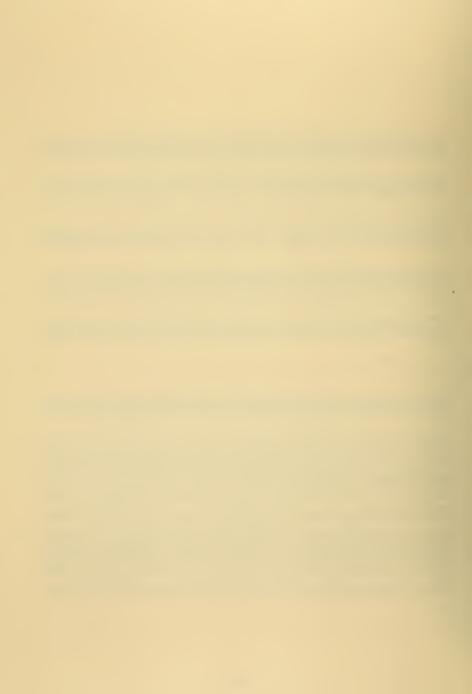
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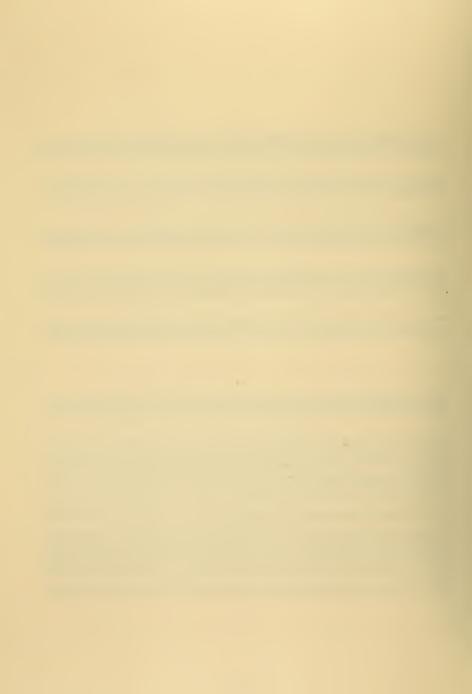




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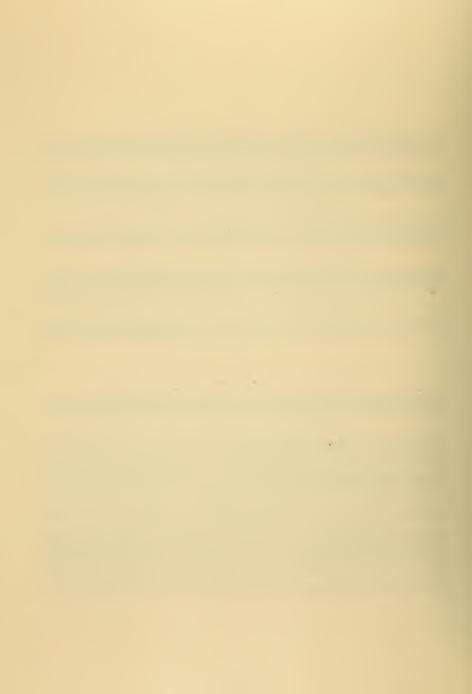






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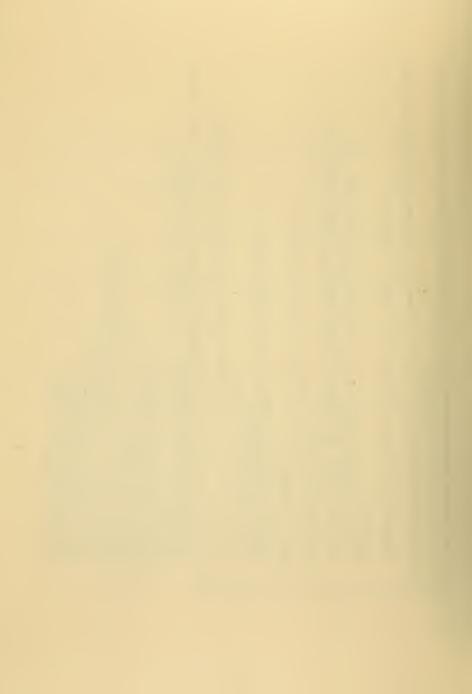


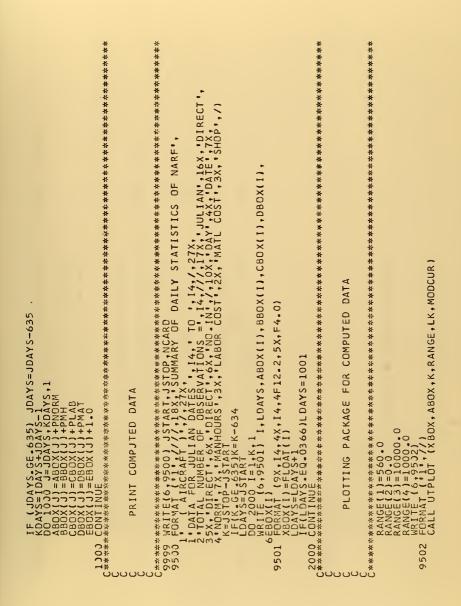
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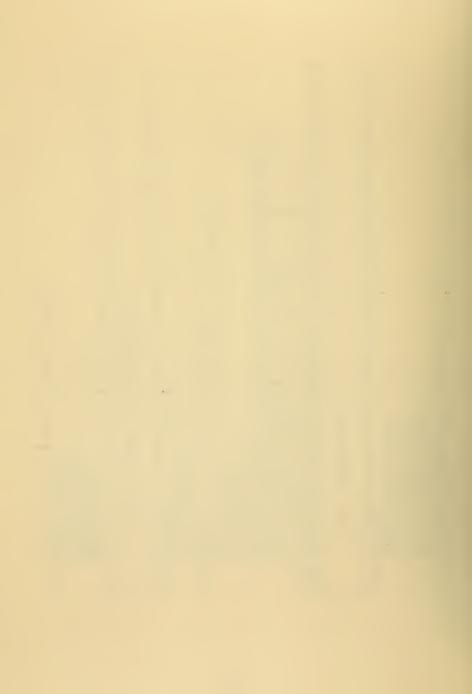
(1) ALL MANHOURS AND COSTS ARE UNIFORMLY DISTRIBUTED OVER THE # OF DAYS IN THE SHOP

(2) WORK BEGINS ON ENTRANCE DATE BUT NO WORK IS DONE ON EXIT DAY.

(3) A SEVEN DAY WORK WEEK IS ASSUMED FOR COMPUTATION PURPOSES. DATA CARDS ARE TO BE IN STANDARD FORMAT PREPARED FOR THESIS WORK EARLIEST ENTRANCE JULIAN DATE (ND EARLIER THAN CY 1970 LATEST EXIT JULIAN DATE (ND LATER THAN CY 1971) NUMBER OF DATA CARDS PORMAT (2x,14,2x,14,2x,14). NARF (AIRCRAFT SUB-ROUTINES REQUIRED TALLY AND UTPLOT, BOTH PROGRAMS LISTED IN APPENDIX COMPUTATION OF DAILY STATISTICS FOR 8 PPENDIX INPUTS REQUIRED 0006 9001







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WRITE (6,9503)

WRITE (6,9503)

WRITE (6,9503)

WRITE (6,9502)

WRITE (6,9503)

WRITE (6,9503)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              (///,23x, GRAPH OF NO. JOBS IN SHOP

X AXIS 1 FOR AIRCRAFT')

I=140,420,1
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(3)=140.0
(3)=9500.0
(4)=6500.0
(7)=6500.0
(6,9502)
(6,9503)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              X BOX, BBOX, K, RANGE, LK, MODCUR
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 ANGE (4) = 139

ANGE (4) = 880X(1)

ANGE (1) = 880X(1)

ANGE (2) = 11

ANGE (4) = 12

ANGE (4) =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      XXXXXX | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000 | 13000
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WRITE (6,9504)
RANGE (3)=65000.0
RANGE (4)=35000.0
WRITE (6,9502)
RANGE (3)=27000.0
RANGE (3)=27000.0
RANGE (4)=15000.0
WRITE (6,9502)
WRITE (6,9502)
WRITE (6,9502)
WRITE (6,9502)
RANGE (4)=09.0
WRITE (6,9502)
RANGE (4)=00.0
RANGE (4)=00.0
WRITE (6,9502)
WRITE (6,9502)
WRITE (6,9502)
WRITE (6,9503)
WRITE (6,9503)
WRITE (6,9503)
WRITE (6,9503)
ERROR
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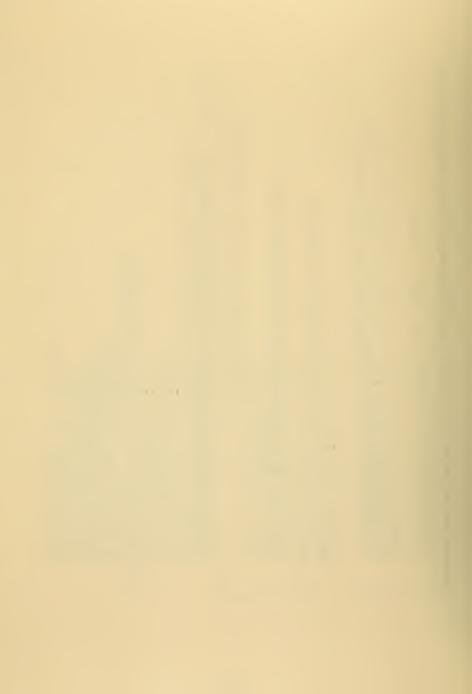
DAY ON EXIT (OVER TH THESIS EARLIEST ENTRANCE JULIAN DATE (NO EARLIER THAN CY 1970 LATEST EXIT JULIAN DATE (NO LATER THAN CY 1971) NUMBER OF DATA CARD FORMAT (2x,14,2x,14,2x,14). NO WORK IS DONE FOR COMPUTATION SUMPTIONS

(1) ALL MANHOURS AND COSTS ARE UNIFORMLY DISTRIBUTED

OF DAYS IN THE SHOP

(2) WORK BEGINS ON ENTRANCE DATE BUT NO WORK IS DONE

(3) A SEVEN DAY WORK WEEK IS ASSUMED FOR COMPUTATION FOR a ENGINES IN APPENDIX TO BE IN STANDARD FORMAT PREPARED NARF SUBROUTINES REQUIRED: TALLY AND UTPLOT, BOTH PROGRAMS LISTED FOR DAILY STATISTICS P CARDS ARE INPUTS REQUIRED OMPUTATION DATA 1006 **社社社社** 0006 0



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9999 WRITE(6,9500) JSTART.JSTOP.NCARD

9500 FORMAT ('1'.'//',18X'.SUMMARY OF DAILY STATISTICS OF NARF',

1 TE NOTAL NUMBER OF OBSERVATIONS = '14,//',17X'.JULIAN',16X'.DIRECT',

2 TOTAL NUMBER OF OBSERVATIONS = '14,///',17X'.JULIAN',16X'.DIRECT',

35X'.DIRECT',6X'.DIRECT',6X'.NO.IN'/',10X'.DAY',4X'.DATE',7X'.

4 NORN'',7X'.MANHOURS',3X','LABOR COST',2X',MATL COST',3X'.SHOP',/)

K JSTOP - 055K K - 634

LOAYS = JSTART

LOAYS = JSTART

LOAYS = JSTART

LOAYS = LOAYS + 14,4X'.14,4F12.2,5X'.F4.0)

SOON (I) = LOAT(I) = LOAT(I) = LOAT(I) = LOAYS + 10.0

CONTINUE CONTINUE CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Y AXIS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          PRORATED NORM
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  RANGE(1) = 480.0

RANGE(2) = 0.0

RANGE(4) = 0.0

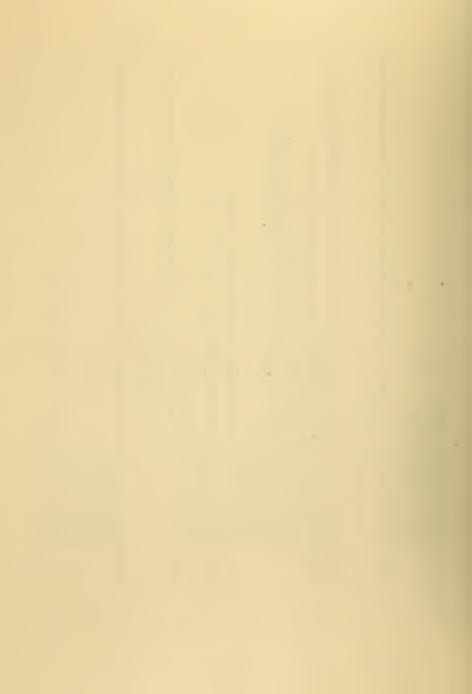
WRITE(6) = 50.0

RANGE(4) = 0.0

RANGE(5) = 0.0

RANGE(7) + 2.0

RANGE(7) + 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             PLOTTING PACKAGE FOR COMPUTED DATA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   PRINT COMPUTED DATA
DD00 JBDAY S+
BBOX(J) = BBOX(J) +
CBDX(J) = BBOX(J) +
CBDX(J) = CBDX(J) +
CBDX(J) +
CBDX(J) = CBDX(J) +
CBDX(J) +
CBDX(J) = CBDX(J) +
CBDX(J) +
CB
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```
6.0AYS ( x AXIS ) FOR ENGINES')
WRITE (6,9502)
CALL UTPLOT ( X AXIS )
FOR WRITE (6,9504)
WRITE (6,9504)
WRITE (6,9504)
CALL UTPLOT ( X AXIS )
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WRITE (6,9502)
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RANGE(1)=430.0

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x AXIS ) FOR ENGINES')
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PLOT (XBOX,BBOX,K,RANGE,LK,MODCUR)
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RANGE (4)=13000.0

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WRITE (6.9502)=170.0

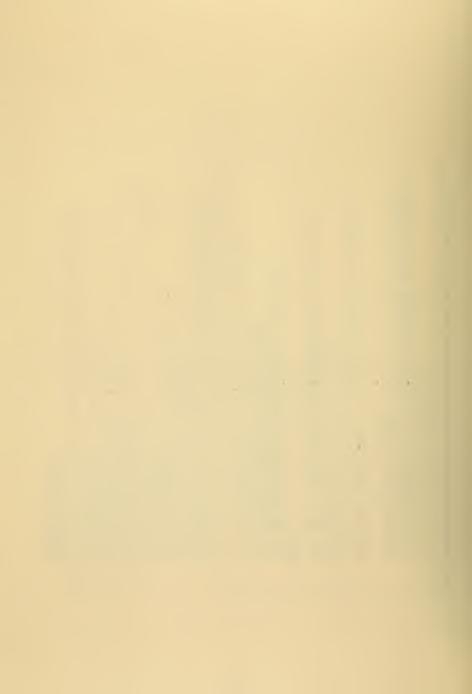
RANGE (4)=80.0

WRITE (6.9502)

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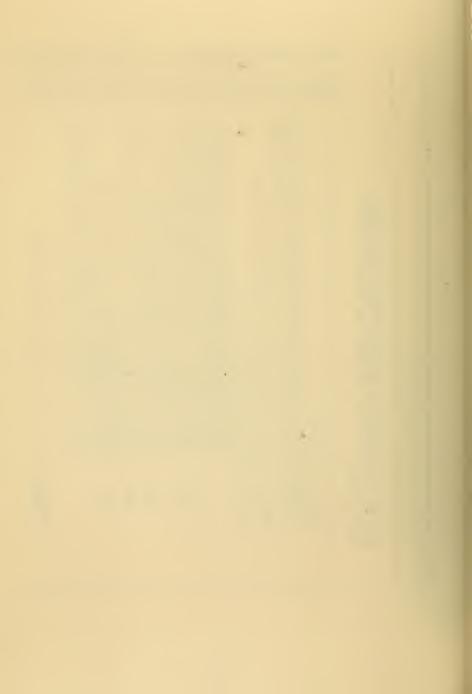


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DATA S'NV', IER, LK, MODCUR/16JO*1.0, 1.0, 1.0,
DATA S'NV', IER, LK, MODCUR/16JO*1.0, 1.0, 1.0,
DATA Z'BOX/500% 0.0,
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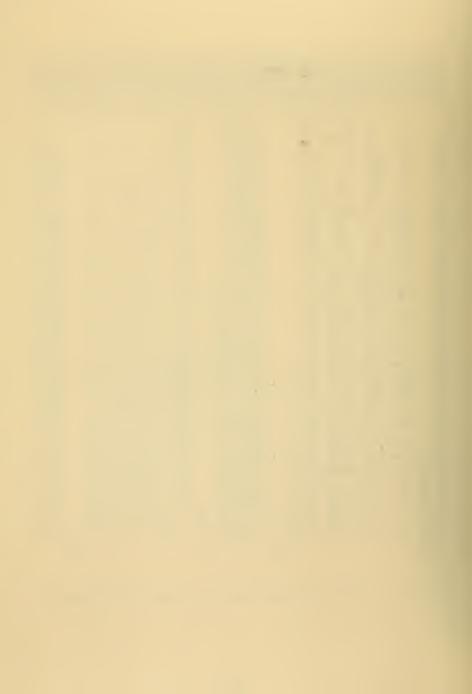
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APPENDIX E. LAGRANGE COMPUTATIONS

Minimize the budget subject to production function constraints.

MINIMIZE
$$\mathbf{B} = P^{L}(L_{1}+L_{2}+L_{3}) + P^{M}(M_{1}+M_{2}+M_{3})$$

$$+ M^{P}(P_{1}+P_{2}+P_{3})$$
S.T. $Q_{1} = a_{1}L_{1}^{\alpha_{1}} M_{1}^{\beta_{1}} P_{1}^{\gamma_{1}}$

$$Q_{2} = a_{2}L_{2}^{\alpha_{2}} M_{2}^{\beta_{2}} P_{2}^{\gamma_{2}}$$

$$Q_{3} = a_{3}L_{3}^{\alpha_{3}} M_{3}^{\beta_{3}} P_{3}^{\gamma_{3}}$$

Lagrange Equation:

$$= P^{L}(L_{1}+L_{2}+L_{3})+P^{M}(M_{1}+M_{2}+M_{3})+P^{P}(P_{1}+P_{2}+P_{3})$$

$$- \lambda_{1}(Q_{1}-a_{1}L_{1}^{\alpha_{1}}M_{1}^{\beta_{1}}P_{1}^{\gamma_{1}})-\lambda_{2}(Q_{2}-a_{2}L_{2}^{\alpha_{2}}M_{2}^{\beta_{2}}P_{2}^{\gamma_{2}})$$

$$- \lambda_{3}(Q_{3}-a_{3}L_{3}^{\alpha_{3}}M_{3}^{\beta_{3}}P_{3}^{\gamma_{3}})$$



STEP 1 By taking the partial derivative with respect to each of the resources and λ , twelve simultaneous equations can be written.

$$(1) \quad \frac{\partial L_{1}}{\partial L_{1}} = P^{L} + \lambda_{1} a_{1} \alpha_{1} L_{1}^{\alpha_{1} - 1} M_{1}^{\beta_{1}} P_{1}^{\gamma_{1}} = 0$$

(2)
$$\frac{\partial L_2}{\partial L_2} = P^L + \lambda_2 a_2 \alpha_2 L_2^{\alpha_2 - 1} M_2^{\beta_2} P_2^{\gamma_2} = 0$$

(3)
$$\frac{\partial L_3}{\partial L_3} = P^L + \lambda_3 a_3 \alpha_3 L_3^{\alpha_3 - 1} M_3^{\beta_3} P_3^{\gamma_3} = 0$$

$$(4) \quad \frac{\partial L}{\partial M_{1}} = P^{M} + \lambda_{1} a_{1} \beta_{1} L_{1}^{\alpha} M_{1}^{\beta} 1^{-1} P_{1}^{\gamma} 1 = 0$$

$$(5) \quad \frac{\partial L}{\partial M_2} = P^{M} + \lambda_2 a_2 \beta_2 L_2^{\alpha_2} M_2^{\beta_2 - 1} P_2^{\gamma_2} = 0$$

(6)
$$\frac{\partial L}{\partial M_3} = P^M + \lambda_3 a_3 \beta_3 L_3^{\alpha_3} M_3^{\beta_3 - 1} P_3^{\gamma_3} = 0$$

$$(7) \quad \frac{\partial L}{\partial P_{1}} = P^{P} + \lambda_{1} a_{1} \gamma_{1} L_{1}^{\alpha_{1}} M_{1}^{\beta_{1}} P_{1}^{\gamma_{1}-1} = 0$$

(8)
$$\frac{\partial L}{\partial P_2} = P^P + \lambda_2 a_2 \gamma_2 L_2^{\alpha_2} M_2^{\beta_2} P_2^{\gamma_2 - 1} = 0$$

$$(9) \quad \frac{\partial L}{\partial P_3} = P^P + \lambda_3 a_3 \gamma_3 L_3^{\alpha} M_3^{\beta} P_3^{\gamma} P_3^{-1} = 0$$

$$(10) \quad \frac{\partial L}{\partial \lambda_1} = 0 \quad \stackrel{\text{IFF}}{<=>} \quad Q_1 = a_1 L_1^{\alpha_1} M_1^{\beta_1} P_1^{\gamma_1}$$



(11)
$$\frac{\partial L}{\partial \lambda_2} = 0$$
 IFF $Q_2 = a_2 L_2^{\alpha} 2 M_2^{\beta} 2 P_2^{\gamma} 2$

(12)
$$\frac{\partial L}{\partial \lambda_3} = 0$$
 $\stackrel{\text{IFF}}{\Leftarrow} Q_3 = a_3 L_3^{\alpha} M_3^{\beta} P_3^{\gamma}$

STEP 2. Put into equations (1) to (9) the values of Q_1 , Q_2 , Q_3 from equations (10), (11) and (12).

Equations (1) to (9) are rewritten as follows:

$$(1) \quad P^{L} + \frac{\lambda_{1}\alpha_{1}Q_{1}}{L_{1}} = 0$$

(2)
$$P^{L} + \frac{\lambda_2 \alpha_2 Q_2}{L_2} = 0$$

(3)
$$P^{L} + \frac{\lambda_{3}^{\alpha_{3}Q_{3}}}{L_{3}} = 0$$

$$(4) \quad P^{M} + \frac{\lambda_{1}\beta_{1}Q_{1}}{M_{1}} = 0$$

(5)
$$P^{M} + \frac{\alpha_2^{\beta_2^{Q}}}{M_2^{Q}} = 0$$

(6)
$$P^{M} + \frac{\lambda_{3}^{\beta_{3}}Q_{3}}{M_{3}} = 0$$

$$(7) \quad P^{P} + \frac{\lambda_{1} \gamma_{1} Q_{1}}{P_{1}} = 0$$

(8)
$$P^{P} + \frac{\lambda_{2} \gamma_{2} Q_{2}}{P_{2}} = 0$$

(9)
$$P^{P} + \frac{\lambda_{3} \gamma_{3} Q_{3}}{P_{3}} = 0$$



STEP 3. The λ 's are in this case the shadow prices and are here isolated to look at equal expressions.

$$(1) \quad -\frac{L_1 P^L}{\alpha_1 Q_1} = \lambda_1$$

$$(4) \quad -\frac{M_1 P^M}{\beta_1 Q_1} = \lambda_1$$

$$(7) \quad \frac{P_1 P^P}{\gamma_1 Q_1} = \lambda_1$$

$$(2) \quad -\frac{L_2 P^L}{\alpha_2 Q_2} = \lambda_2$$

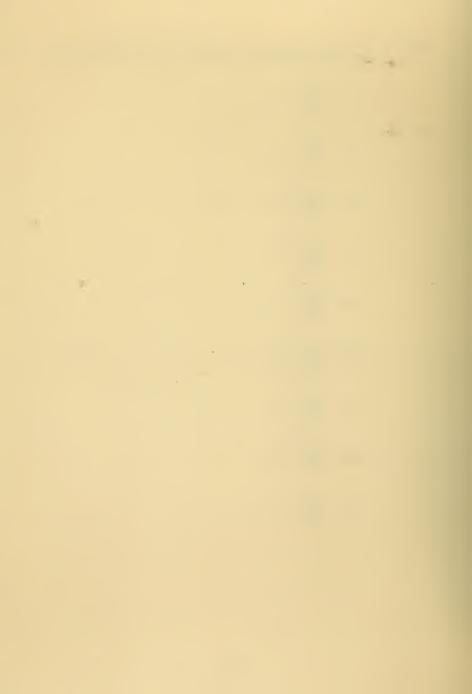
$$(5) \quad \frac{M_2 P^M}{\beta_2 Q_2} = \lambda_2$$

$$(8) \quad -\frac{P_2 P^P}{\gamma_2 Q_2} = \lambda_2$$

$$(3) \quad -\frac{L_3 P^L}{\alpha_3 Q_3} = \lambda_3$$

$$(6) \quad -\frac{M_3 P^M}{\beta_3 Q_3} = \lambda_3$$

$$(9) \quad -\frac{P_3 P^P}{\gamma_3 Q_3} = \lambda_3$$



STEP 4. Eliminate output variables in order to look at relationships between resources.

(I) (1) and (4)
$$-\frac{\mathbf{L}_{1}\mathbf{P}^{L}}{\alpha_{1}\mathbf{Q}_{1}} = -\frac{\mathbf{M}_{1}\mathbf{P}^{M}}{\beta_{1}\mathbf{Q}_{1}}, \quad \frac{\mathbf{P}^{L}}{\mathbf{P}^{M}} = \frac{\mathbf{M}_{1}\alpha_{1}}{L_{1}\beta_{1}}$$

(II) (1) and (7)
$$-\frac{L_1P^L}{\alpha_1Q_1} = -\frac{P_1P^P}{\gamma_1Q_1}$$
, $\frac{P^L}{P^P} = \frac{P_1\alpha_1}{L_1\gamma_1}$

(III) (4) and (7)
$$-\frac{M_1P^M}{\beta_1Q_1} = -\frac{P_1P^P}{\gamma_1Q_1}$$
, $\frac{P^M}{P^P} = \frac{P_1\beta_1}{M_1\gamma_1}$

By following the same method with (2),(3),(5),(6), (8),(9) the following relationships are also true:

(IV)
$$\frac{P^{L}}{P^{M}} = \frac{M_2 \alpha_2}{L_2 \beta_2}$$

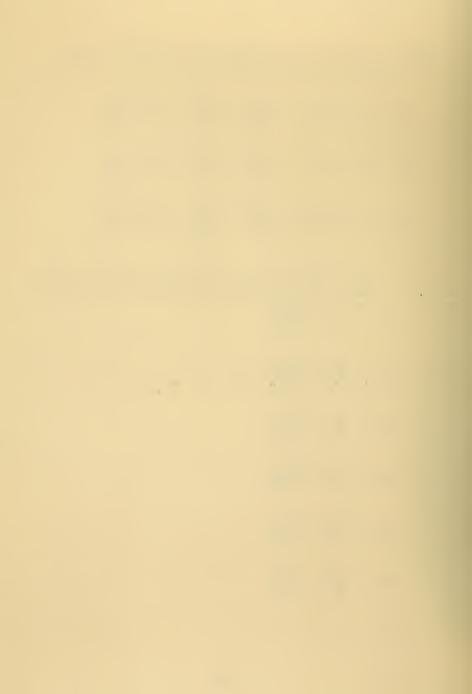
$$(V) \qquad \frac{P^{L}}{P^{P}} = \frac{P_{2}\alpha_{2}}{L_{2}\gamma_{2}}$$

(VI)
$$\frac{P^{M}}{P^{P}} = \frac{P_{2}\beta_{2}}{M_{2}\gamma_{2}}$$

(VII)
$$\frac{P^{L}}{P^{M}} = \frac{{}^{M}3^{\alpha}3}{{}^{L}3^{\beta}3}$$

(VIII)
$$\frac{P^{L}}{P^{P}} = \frac{P_{3}^{\alpha} 3}{L_{3}^{\gamma} 3}$$

(IX)
$$\frac{P^{M}}{P^{P}} = \frac{P_{3}^{\beta}3}{M_{3}^{\gamma}3}$$



STEP 5. Substitute resources in the production function to arrive at outputs in terms of a single resource. One example will show the method and the results can be extrapolated to all production functions. Look at the aircraft production function in terms of labor only.

$$Q_1 = aL_1^{\alpha_1}M_1^{\beta_1}P_1^{\gamma_1}$$

 M_{γ} in terms of L_{γ} from eq. (I)

$$M_{1} = \frac{\beta_{1} P^{L} L_{1}}{\alpha_{1} P^{M}}$$

 P_1 in terms of L_1 from eq. (II)

$$P_1 = \frac{\gamma_1 P^{L_1}}{\alpha_1 P^{P}}$$

$$Q_{1} = a_{1}L_{1}^{\alpha_{1}} \left(\frac{\beta_{1}P^{L}L_{1}}{\alpha_{1}P^{M}}\right)^{\beta_{1}} \left(\frac{\gamma_{1}P^{L}L_{1}}{\alpha_{1}P^{P}}\right)^{\gamma_{1}}$$

$$Q_{1} = a_{1} \left(\frac{\beta_{1}^{P^{L}}}{\alpha_{1}^{P^{M}}}\right)^{\beta_{1}} \left(\frac{\gamma_{1}^{P^{L}}}{\alpha_{1}^{P^{P}}}\right)^{\gamma_{1}} L_{1}^{(\frac{\alpha}{\alpha}_{1} + \frac{\beta}{\alpha}_{1} + \frac{\gamma}{\alpha}_{1})}$$

Since $a,\alpha,\beta,\gamma,P^L,P^M,P^P$ are all constants this expression can be rewritten as

$$Q_1 = AL_1^u \qquad \text{where } A = a_1 \left(\frac{\beta_1 P^L}{\alpha_1 P^M}\right)^{\beta_1} \left(\frac{\gamma_1 P^L}{\alpha_1 P^P}\right)^{\gamma_1} \qquad \text{and} \qquad \qquad u = \alpha_1 + \beta_1 + \gamma_1$$



 \boldsymbol{Q}_{1} in terms of material will be represented by

$$Q_1 = BM_1^U \qquad \text{where } B = a_1 (\frac{\beta_1 P^L}{\alpha_1 P^M})^{\alpha_1} (\frac{\gamma_1 P^M}{\beta_1 P^P})^{\gamma_1}$$
 and $u = \alpha_1 + \beta_1 + \gamma_1$

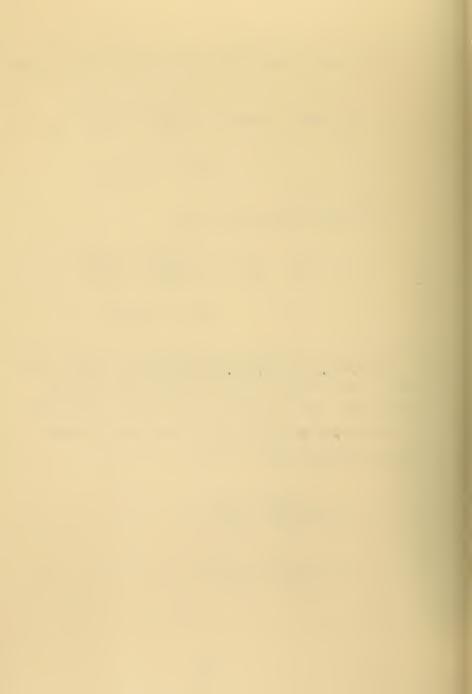
Q, in terms of pipeline will be:

$$Q_1 = CP_1^U$$
 where $C = a_1 \left(\frac{\alpha_1 P^P}{\gamma_1 P^L}\right)^{\alpha_1} \left(\frac{\gamma_1 P^M}{\beta_1 P^P}\right)^{\beta_1}$
and $u = \alpha_1 + \beta_1 + \gamma_1$

This method will give output equations in terms of each input. The constants for Q_2 and Q_3 are D,E,F and G,H,J respectively. The values for these constants can be found in the same manner as A,B, and C. Constants for engine output expressions are:

$$D = a_2 \left(\frac{\beta_2 P^L}{\alpha_2 P^M}\right)^{\beta_2} \left(\frac{\gamma_2 P^L}{\alpha_2 P^P}\right)^{\gamma_2}$$

$$E = a_2 \left(\frac{\beta_2 P^L}{\alpha_2 P^M}\right)^{\alpha_2} \left(\frac{\gamma_2 P^M}{\beta_2 P^P}\right)^{\gamma_2}$$



$$F = a_2 \left(\frac{\alpha_2 P^P}{\gamma_2 P^L}\right)^{\alpha_2} \left(\frac{\gamma_2 P^M}{\beta_2 P^P}\right)^{\beta_2}$$

The constants D,E, and F allow the equations for engine output to be written as follows:

$$Q_2 = DL_2^2$$

$$Q_2 = EM_2^V$$

$$Q_2 = FP_2^V$$
 where $V = \alpha_2 + \beta_2 + \gamma_2$

constants for component output are:

$$G = a_3 \left(\frac{\beta_3 P^L}{\alpha_3 P^M}\right)^{\beta_3} \left(\frac{\gamma_3 P^L}{\alpha_3 P^P}\right)^{\gamma_3}$$

$$H = a_3 \left(\frac{\beta_3 P^L}{\alpha_3 P^M}\right)^{\alpha_3} \left(\frac{\gamma_3 P^M}{\beta_3 P^P}\right)^{\gamma_3}$$

$$J = a_{3} (\frac{\alpha_{3}^{P}}{\gamma_{3}^{P}})^{\alpha_{3}} (\frac{\gamma_{3}^{P}}{\beta_{3}^{P}})^{\beta_{3}}$$



the component equations become

$$Q_3 = GL_3^W$$

$$Q_3 = HM_3^W$$

$$Q_3 = JP_3^W$$
 where $W = \alpha_3 + \beta_3 + \gamma_3$

STEP 6. Using the relationships found in step 5, the budget function is written in terms of output and prices of L, M, and P.

$$\mathbf{B} = \mathbf{P}^{\mathbf{L}} (\mathbf{L}_{1} + \mathbf{L}_{2} + \mathbf{L}_{3}) + \mathbf{P}^{\mathbf{M}} (\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{M}_{3}) + \mathbf{P}^{\mathbf{P}} (\mathbf{P}_{1} + \mathbf{P}_{2} + \mathbf{P}_{3})$$

$$\mathbf{B} = \mathbf{P}^{\mathbf{L}} [(\frac{Q_{1}}{A})^{\frac{1}{u}} + (\frac{Q_{2}}{D})^{\frac{1}{v}} + (\frac{Q_{3}}{G})^{\frac{1}{w}}]$$

$$+ \mathbf{P}^{\mathbf{M}} [(\frac{Q_{1}}{B})^{\frac{1}{u}} + (\frac{Q_{2}}{E})^{\frac{1}{v}} + (\frac{Q_{3}}{H})^{\frac{1}{w}}]$$

$$+ \mathbf{P}^{\mathbf{P}} [(\frac{Q_{1}}{G})^{\frac{1}{u}} + (\frac{Q_{2}}{H})^{\frac{1}{v}} + (\frac{Q_{3}}{H})^{\frac{1}{w}}]$$

or

$$\mathbf{B} = (\frac{P^{L}}{A^{u}} + \frac{P^{M}}{B^{u}} + \frac{P^{P}}{C^{u}})Q_{1}^{\frac{1}{u}} + (\frac{P^{L}}{D^{v}} + \frac{P^{M}}{D^{v}} + \frac{P^{P}}{F^{v}})Q_{2}^{\frac{1}{v}}$$
$$+ (\frac{P^{L}}{D^{u}} + \frac{P^{M}}{D^{u}} + \frac{P^{M}}{D^{u}} + \frac{P^{P}}{D^{u}})Q_{3}^{\frac{1}{w}}$$



$$\mathbf{B} = K_1 Q_1^{\frac{1}{u}} + K_2 Q_2^{\frac{1}{v}} + K_3 Q_3^{\frac{1}{w}}$$

where \mathbf{K}_1 , \mathbf{K}_2 , and \mathbf{K}_3 represent values in parentheses from previous equation.



APPENDIX F. CURVE DERIVATION

The budget =
$$TB = K_1Q_1^{\frac{1}{u}} + K_2Q_2^{\frac{1}{v}}$$

Rewrite the equation:

$$Q_{2}^{\frac{1}{v}} = \frac{\mathbb{B}}{K_{2}} - \frac{K_{1}}{K_{2}} Q_{1}^{\frac{1}{u}}$$

$$Q_{2} = (\frac{\mathbb{B}}{K_{2}} - \frac{K_{1}}{K_{2}} Q_{1}^{\frac{1}{u}})^{v}$$

First Derivative:

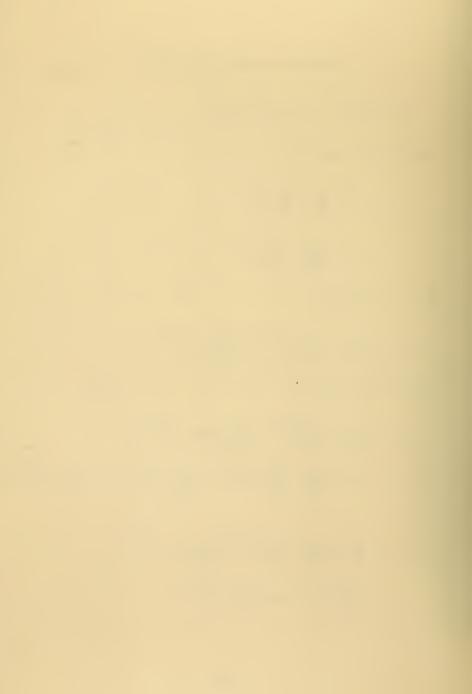
$$Q_2' = [v(\frac{B}{K_2} - \frac{K_1}{K_2} Q_1^{\frac{1}{u}})^{v-1}][-\frac{K_1}{K_2} \frac{1}{u} Q_1^{\frac{1}{u}}]$$

Second Derivative:

$$\begin{aligned} \mathbb{Q}_{2}'' &= \left[v \left(\frac{\mathbb{B}}{K_{2}} - \frac{K_{1}}{K_{2}} \, \mathbb{Q}^{\frac{1}{u}} \right)^{v-1} \left(- \frac{K_{1}}{K_{2}} \, \frac{1}{u} \, \frac{1-u}{u} \, \mathbb{Q}_{1}^{\frac{1}{u}} \, ^{-2} \right) \right] \\ &+ v (v-1) \left(\frac{\mathbb{B}}{K_{2}} - \frac{K_{1}}{K_{2}} \, \mathbb{Q}_{1}^{\frac{1}{u}} \right)^{v-2} \left(- \frac{K_{1}}{K_{2}} \, \frac{1}{u} \, \mathbb{Q}_{1}^{\frac{1}{u}} - 1 \right) \left(- \frac{K_{1}}{K_{2}} \, \frac{1}{u} \, \mathbb{Q}_{1}^{\frac{1}{u}} - 1 \right) \end{aligned}$$

$$Q_{2}" = v \frac{K_{1}}{K_{2}} \frac{1}{u} \left(\frac{B}{K_{2}} - \frac{K_{1}}{K_{2}} Q_{1}^{\frac{1}{u}}\right)^{v-2} \left[\frac{B}{K_{2}} \frac{u-1}{u} Q_{1}^{\frac{1}{u}} - 2\right]$$

$$+ \left(\frac{K_{1}}{K_{2}} \frac{1-u}{u} + (v-1) \frac{K_{1}}{K_{2}} \frac{1}{u} Q_{1}^{\frac{2u}{u}} - 2\right)$$



$$Q_{2}" = v \frac{K_{1}}{K_{2}} \frac{1}{u} \left(\frac{B}{K_{2}} - \frac{K_{1}}{K_{2}} Q_{1}^{\frac{1}{u}}\right)^{v-2} \frac{1}{K_{2}} \frac{1}{u} \left[B(u-1)Q_{1}^{\frac{1}{u}} - 2\right]$$

$$+ \left(K_{1}(1-u) + K_{1}(v-1)Q_{1}^{\frac{2}{u}} - 2\right]$$

$$Q_{2}" = vK_{1} \frac{1}{K_{2}^{3}} \frac{1}{u^{2}} (a_{3} - K_{1}Q_{1}^{\frac{1}{u}})^{v-2} Q_{1}^{\frac{1}{u}} - 2 \{B(u-1) + [K_{1}(1-u) + K_{1}(v-1)]Q_{1}^{\frac{1}{u}}\}$$

$$Q_{2}" = \frac{K_{1}}{K_{2}^{3}} \frac{v}{u^{2}} (B-K_{1}Q_{1}^{\frac{1}{u}})^{v-2} Q_{1}^{\frac{1}{u}} - 2[B(u-1)+K_{1}(v-u)Q_{1}^{\frac{1}{u}}]$$

$$> 0$$

$$> 0$$

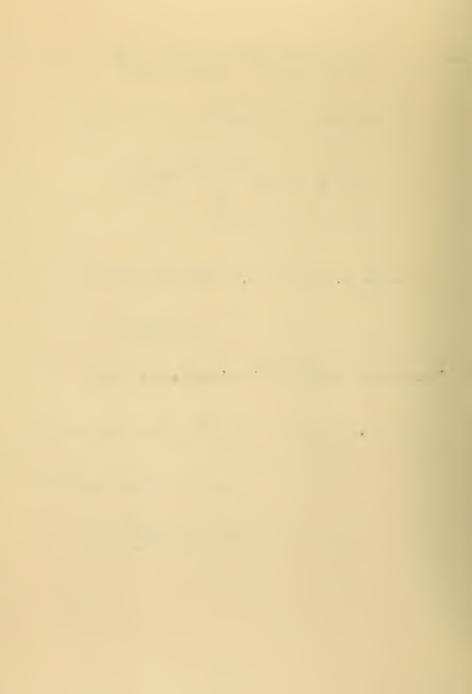
$$= sign dependent on u and v$$

Replace B with $K_1Q_1^{\frac{1}{u}}+K_2Q_2^{\frac{1}{v}}$ in bracketed part of $Q_2^{"}$.

$$[2B(u-1) + K_{1}(v-u)Q_{1}^{\frac{1}{u}}] = [(K_{1}Q_{1}^{\frac{1}{u}} + K_{2}Q_{2}^{\frac{1}{v}})(u-1) + K_{1}(v-u)Q_{1}^{\frac{1}{u}}]$$

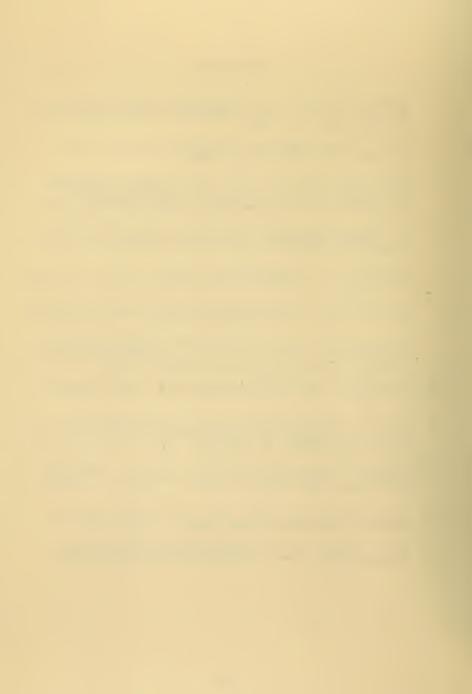
$$= [(K_{1}Q_{1}^{\frac{1}{u}}((u-1) + (v-u)) + K_{2}Q_{2}^{\frac{1}{v}}(u-1)]$$

$$= K_{1}Q_{1}^{\frac{1}{u}}(v-1) + K_{2}Q_{2}^{\frac{1}{v}}(u-1)$$



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The Management System Development Office (MSDO) of the Naval Air Integrated Logistic Support Center (NAILSC) is presently evaluating a prototype system at the Naval Air Rework Facility, North Island (NARFNI). The system is called "work-in-progress inventory control system", (WIPICS). This paper is concerned with that evaluation and shall utilize before WIPICS production data provided by NARFNI for analysis. Three production functions for three different outputs each with three input variables are discussed and inter-relationships are identified and explained. Three Cobb-Douglas productions functions constrain the cost minimization problem. Prices of inputs are constant and a minimum budget is derived for various output levels. Finally, a discussion of the possible results and their meanings is offered to improve under-

standing of the before and after WIPICS analysis.

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